Engineering Efficient Paging Algorithms

Andrei Negoescu

Goethe University Frankfurt

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Joint work with G. Moruz, C. Neumann and V. Weichert

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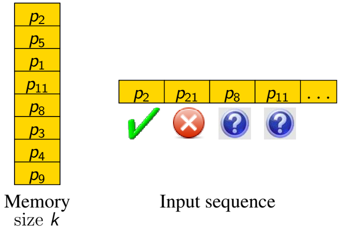
Paging



- Cache of size k
- Page requests online (no information about future)
- Page fault - required page not in memory
  - Bring it in memory; replace some other page
- Minimize # page-faults

Quality Aspects of Paging Strategies

1. **Theoretical worst case guarantees**
   - OPT is the optimal offline solution
   - A is c-competitive: $cost(A) \leq c \cdot cost(OPT) + b$
   - Good algorithms: many

2. **Empirical competitive ratio**
   - Ratio $cost(A)/cost(OPT)$ on real-world traces
   - LRU is good and simple
   - RDM, EELRU, FARL outperform LRU but are more complex

3. Today's talk: **Overall performance**
   - Consider: Page faults and runtime
   - Simple algorithms: LRU (few page faults), FIFO (very fast)
   - We show: RDM is a good alternative

Simple Paging Algorithms

LRU

- Replace the page least recently used
- Competitive ratio: k (optimal)
- Empirical competitive ratio: ≈ 4
- Can be implemented in $O(1)$ per request

FIFO

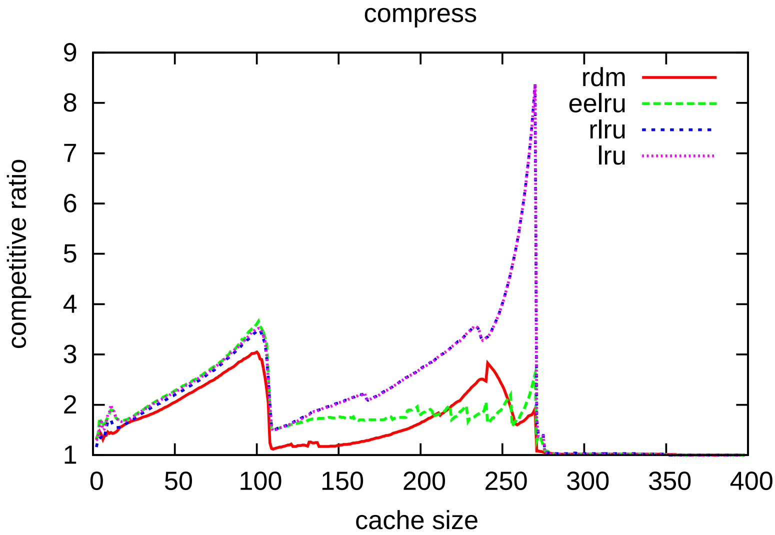
- Replace the page which entered first the cache
- Competitive ratio: k (optimal)
- Empirical performance: worse than LRU
- $O(1)$ per request, faster than LRU

RDM - Main Idea

[Moruz and N. '12]

- Priority assignment for page upon request
- Keeps OPTs cache content, given
  - Sequence seen so far
  - Priority based future prediction
- RDM - Priority assignment
  - Recency: current timestamp
  - Duration: priority $=$ time in OPTs cache
  - Mix: priority $= 0.8 \cdot \textit{recency} + 0.1 \cdot \textit{duration}$

Competitiveness of RDM



RDM - Layer Partition

[Koutsoupias and Papadimitriou '94]

- Tracks cache configurations of optimal solution
- Splits pageset in $k + 1$ disjoint layers $L_0, \ldots, L_k$
  - Implementation: $k + 1$ consecutive time intervals
- OPT caches at most i pages from $L_1 \cup \cdots \cup L_i$
- Update after each page request p
  - Find the layer containing p
  - Merge two consecutive layers
  - Create a new rightmost layer

Example $k = 4$: $\{1, 3, 5\}_0 \quad \{2\}_1 \quad \{6, 8\}_2 \quad \{4\}_3 \quad \{7\}_4$

RDM - Layer Partition

- Three types of requests
  - Revealed – for sure in OPT's cache (rightmost singletons)
  - OPTmiss – for sure not in OPT's cache (L_0)
  - Unrevealed – maybe in OPT's cache (or maybe not)

Example $k = 4$: $\{\boxed{1}, \boxed{3}, \boxed{5}\}_0 \quad \{\boxed{2}\}_1 \quad \{\boxed{6}, \boxed{8}\}_2 \quad \{\boxed{4}\}_3 \quad \{\boxed{7}\}_4$

RDM

- **Cache Update**
  - if $p \in M$ do nothing
  - if $p \notin M$
    - $p \in L_0$: evict page in M having smallest priority
    - $p \in L_i$ and $p \notin M$:
      - identify $j \geq i$ s.t. $|M \cap (L_1 \cup \cdots \cup L_j)| = j$;
      - evict page in this set having smallest priority
- **Layer update** (each request)

- **Runtime**
  - $\Theta(\log k)$ per request using balanced binary trees
  - High constants
  - Implementig it: very slow compared to LRU

Outline

- New compressed layer partition
- A simpler RDM implementation based on arrays
- Runtime experiments
  - RDM: array vs. tree implementation
  - RDM versus LRU and FIFO
  - Overall performance including page fault penalties

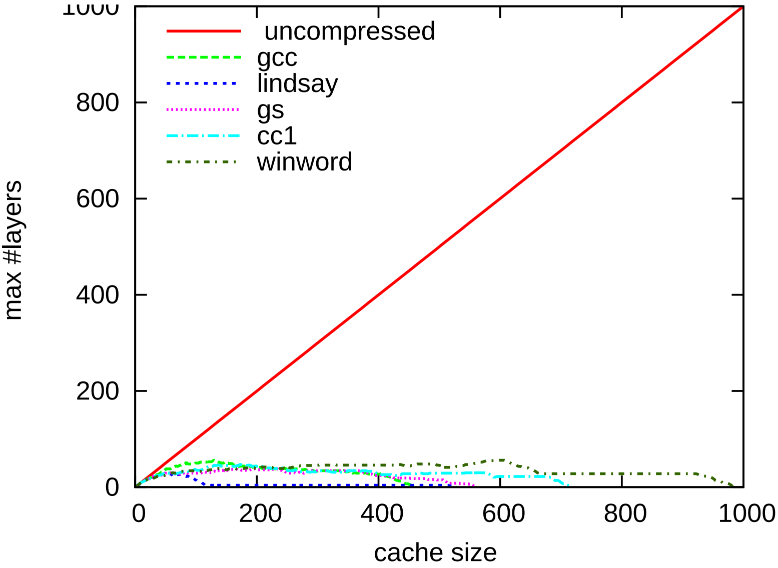
RDM - Compressed Layers

- Observations
  - $> 99\%$ of requests are revealed
  - Layer update faster if we decrease amount of non-empty layers
- New Layer Partition (compressed)
  - Still: OPT caches at most i pages from $L_1 \cup \dots \cup L_i$
  - Difference: No update on revealed requests
  - Difference: $\#$ Non-empty Layers $\leq k + 1$

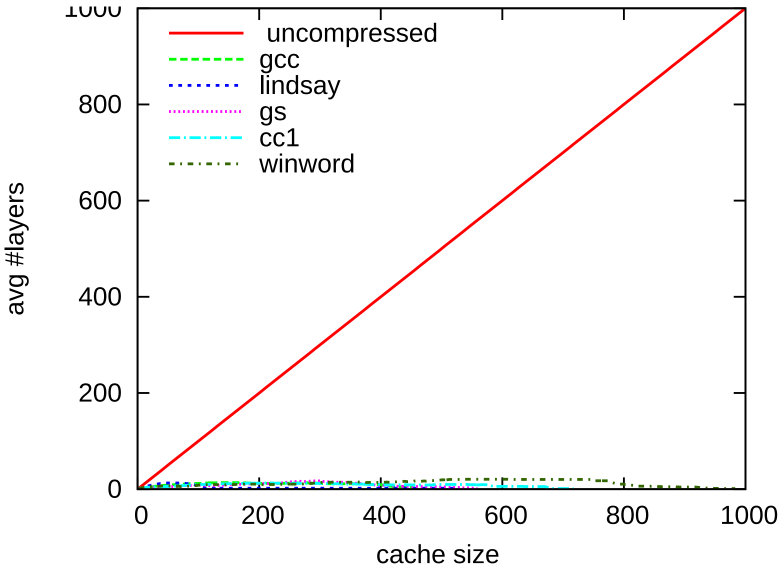
Old: $\{1, 3, 5\}_0 \quad \{2\}_1 \quad \{6, 8\}_2 \quad \{4\}_3 \quad \{7\}_4$

New: $\{1, 3, 5\}_0 \quad \{\}_1 \quad \{2, 6, 8\}_2 \quad \{\}_3 \quad \{4, 7\}_4$

#Non-empty Layers – Max



#Non-empty Layers – Average



RDM - Array Implementation

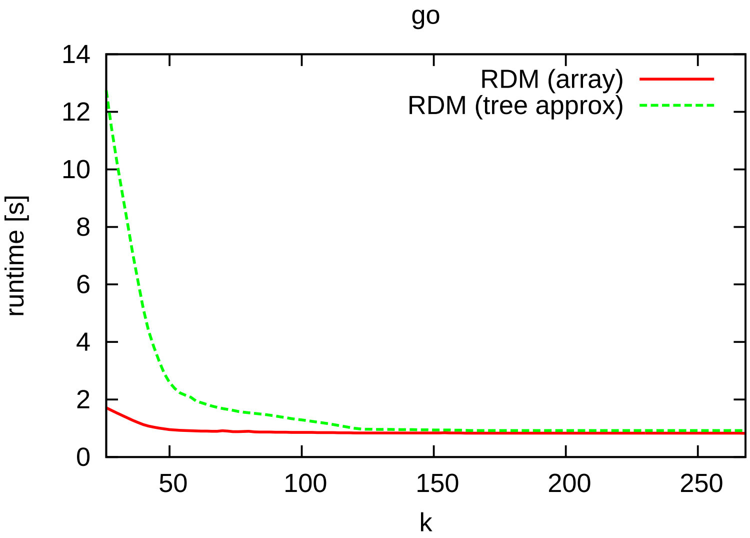
An alternative implementation using arrays

- Store $O(1)$ data per page, timestamp of last request in $O(1)$
- L non-empty layers: interval delimiters in a sorted array
- All operations done by linear traversal

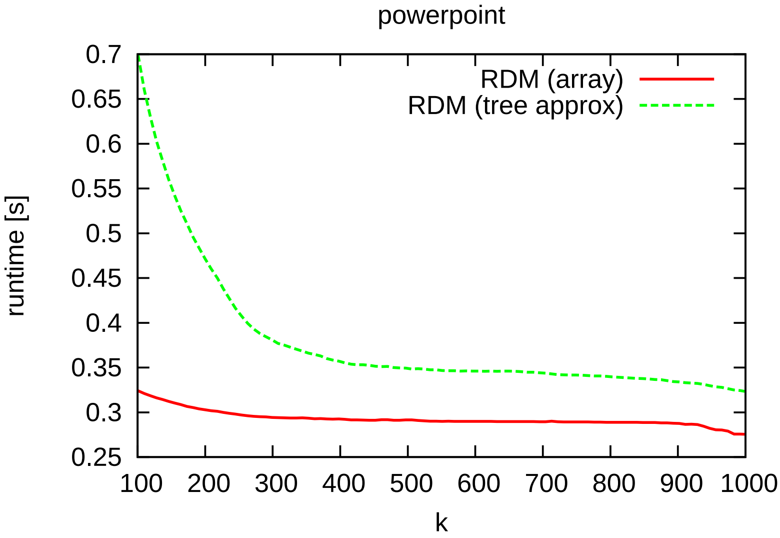
Upon request of page p

- $p \in M$ and revealed: do nothing $> 99\%$ requests $O(1)$
- $p \in M$ and not revealed: Update layers $O(L)$
- $p \notin M$ $O(k)$
  - ...

Runtime RDM implementations



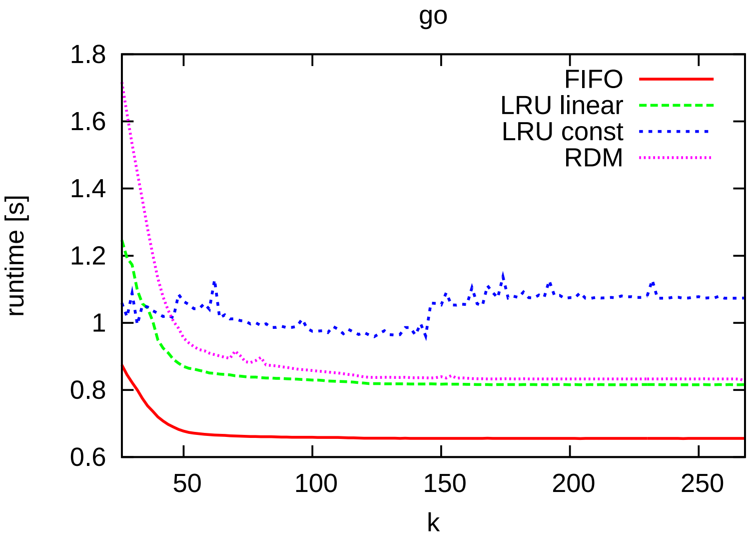
Runtime RDM implementations



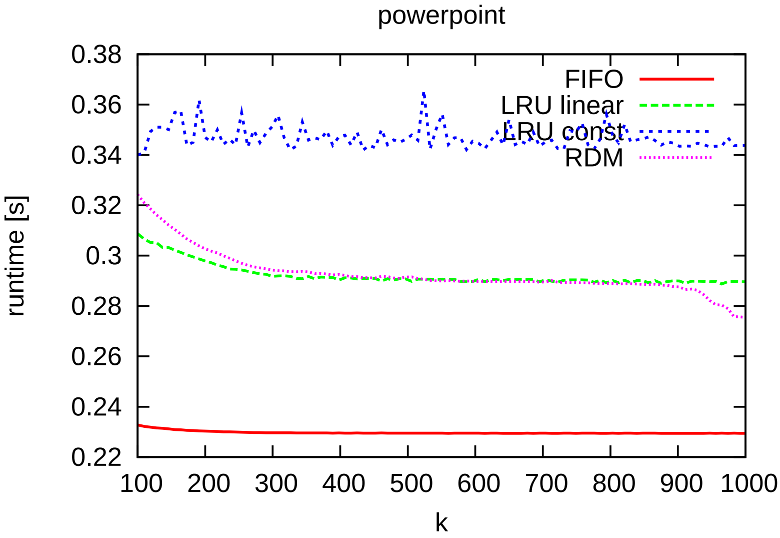
RDM, LRU, and FIFO

- FIFO
  - Circular array
  - $O(1)$ time per page request
- Constant time LRU
  - A recency list – updated for each request
  - $O(1)$ time per page request
- Linear LRU
  - An array storing pages in cache
  - For hits do nothing, misses take $O(k)$ time
- RDM
  - Array version

Runtime RDM vs LRU and FIFO



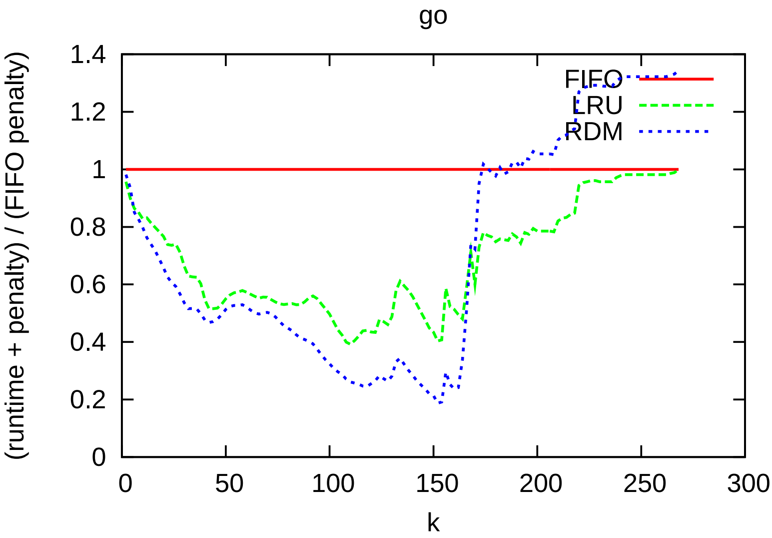
Runtime RDM vs LRU and FIFO



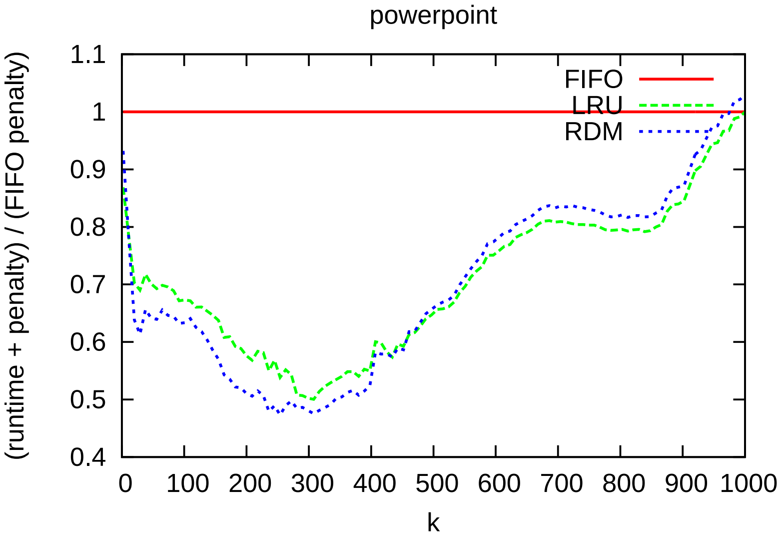
Overall Performance

- Assign page fault a cost of 9 ms
- Penalty $=$ #page faults $\times$ 9ms
- Total runtime $=$ Effective runtime $+$ penalty
- For the algorithms considered:
  - RDM: Effective runtime $+$ penalty
  - FIFO: 0 $+$ penalty
  - LRU: 0 $+$ penalty

Total Runtime (including penalties)



Total Runtime (including penalties)



Conclusions

- Compressed Layers lead to faster implementations
- FIFO is the fastest
- RDM implementation can compete with LRU
- RDM may be of practical interest