A More Reliable Greedy Heuristic for Maximum Matchings in Sparse Random Graphs

Martin Dietzfelbinger[1] Hendrik Peilke[2] Michael Rink[1]

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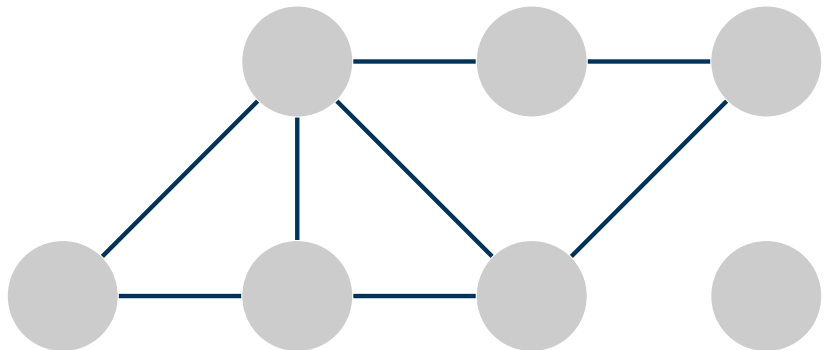
Maximum Cardinality Matching



undirected graph G:

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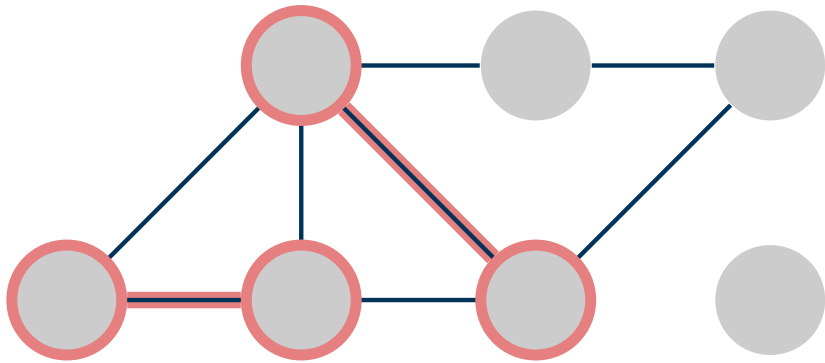

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- A matching M is set of pairwise disjoint edges from G.

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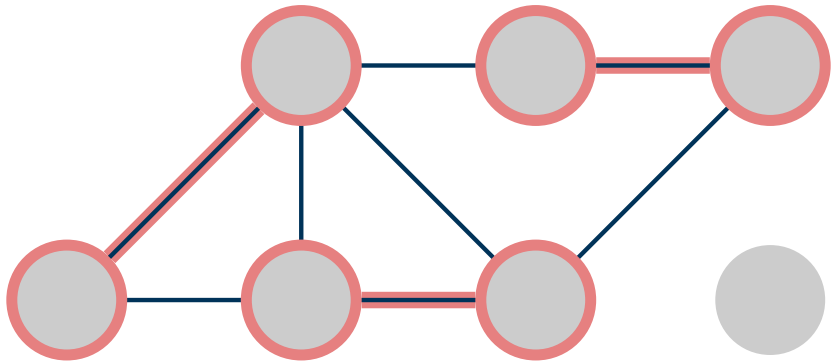

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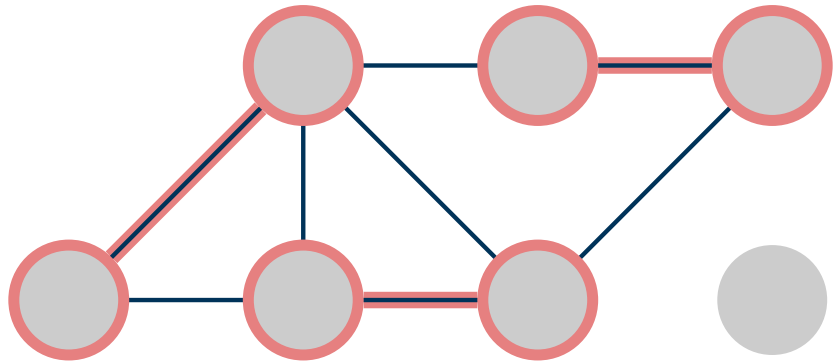

undirected graph G:

- A matching M is set of pairwise disjoint edges from G.
- M is a maximum matching if it has largest possible cardinality.
- The problem of finding a maximum matching is well understood.

Algorithms for Maximum Matchings

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What can be achieved with simpler algorithms?

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  - There are good greedy heuristics with linear running time that are likely to find maximum matchings for a wide range of c.
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Is there a greedy heuristic with no critical region?

Result

We describe a new greedy heuristic with (close to) linear running time and give experimental evidence that this heuristic is likely to find a maximum matching in $G(n; c)$ for all ranges of c.

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- Our approach is motivated by the "selfless algorithm" of [Sanders, 2004] for orienting undirected graphs.
- We compared (quality of solution) our new heuristic with several good heuristics commonly used.

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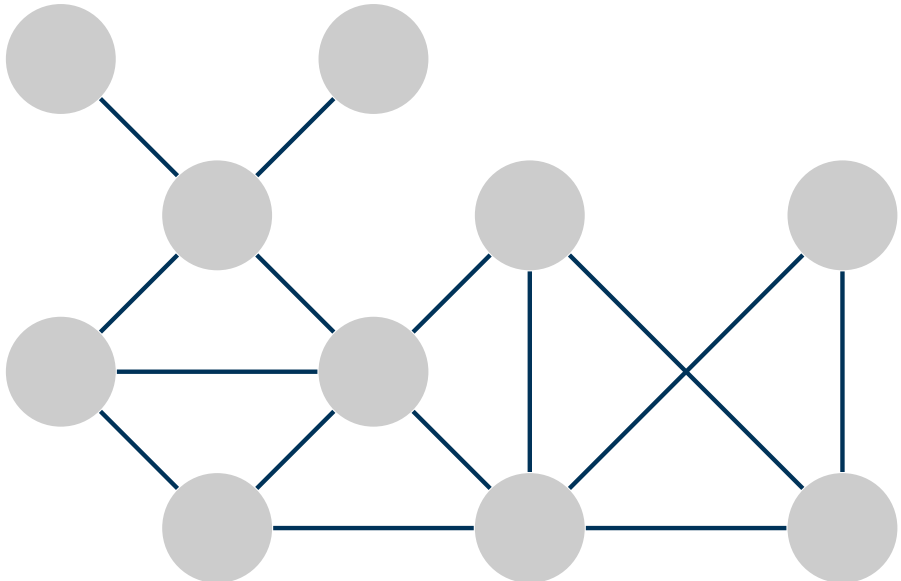
two kinds of reduction steps:

OPT never decrease the size of a maximum matching [Karp and Sipser, 1981]

HEU can decrease the size of a maximum matching

Optimal Reduction Steps



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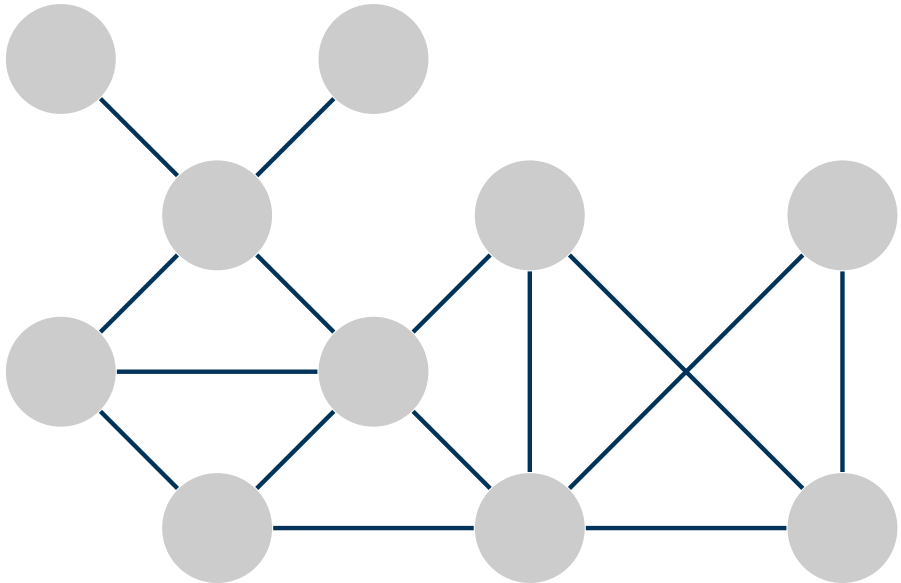
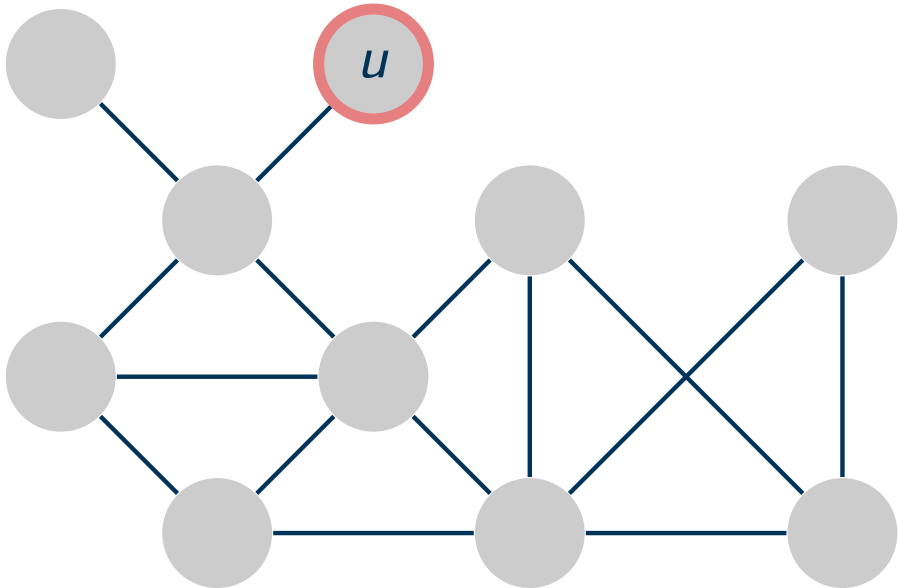


OPT(1)

Optimal Reduction Steps



OPT(1)

- choose node u of degree 1

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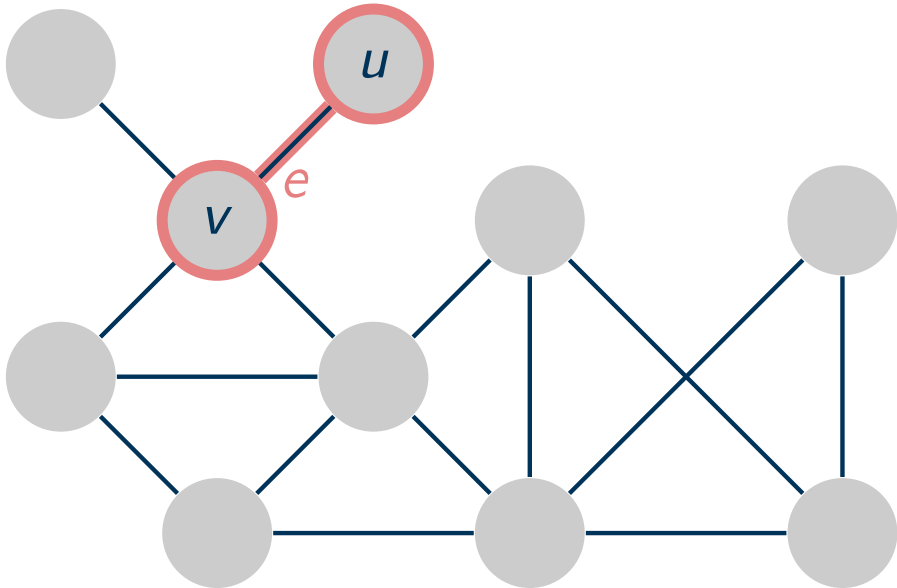

OPT(1)

- choose node u of degree 1
- incident edge $e = \{u, v\}$ belongs to matching

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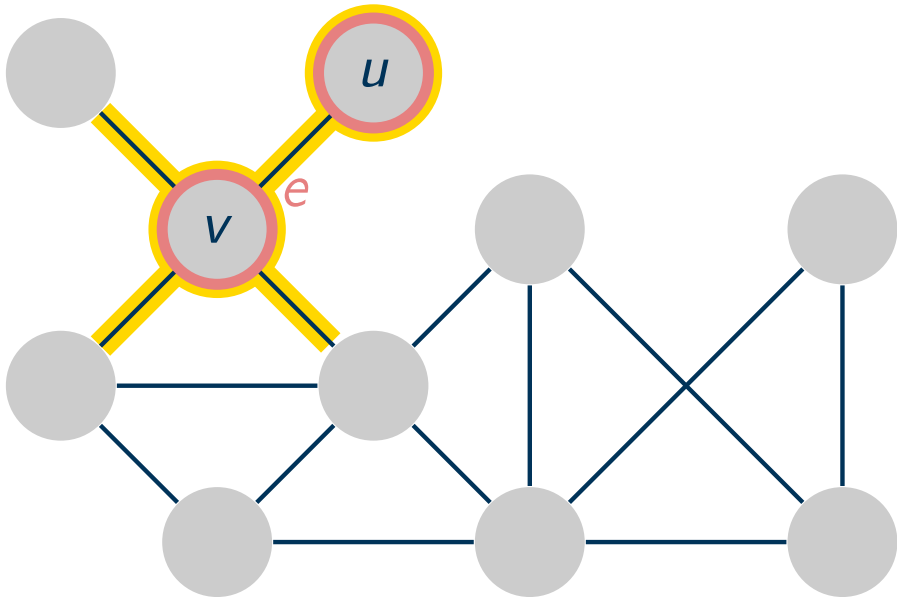

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- shrink graph

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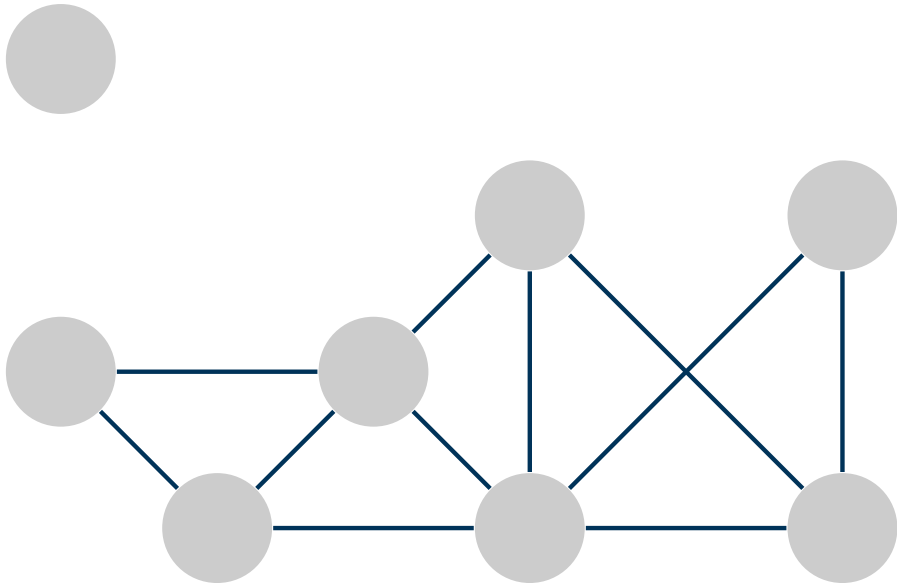

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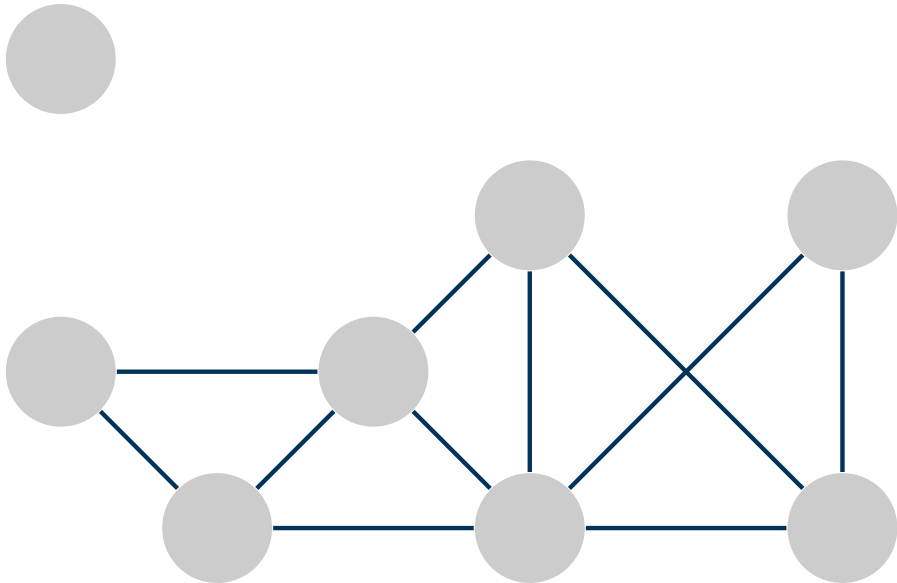

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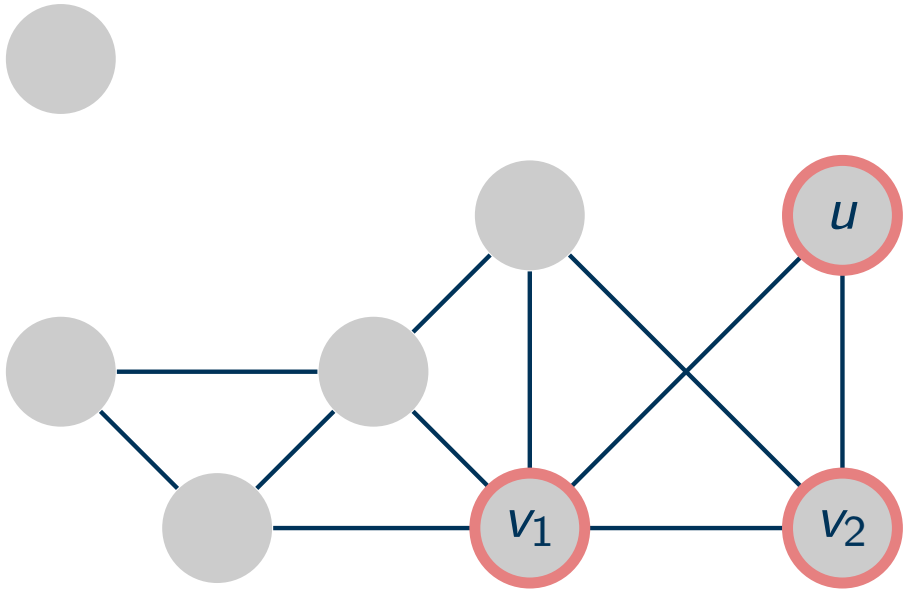

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OPT(2)
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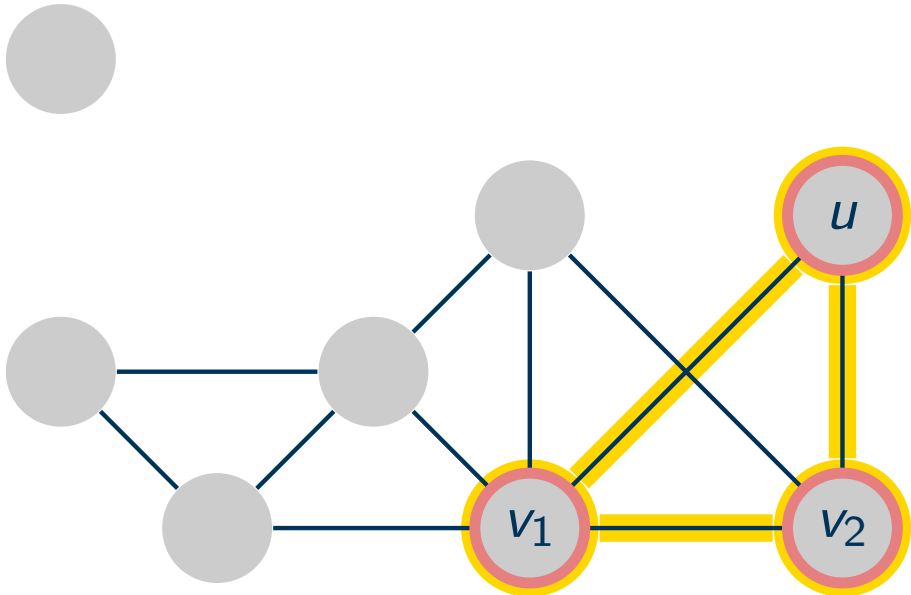

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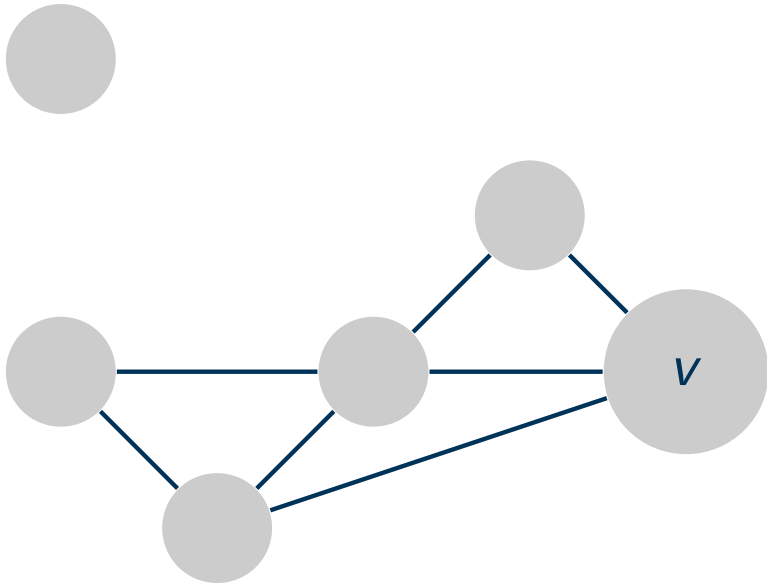

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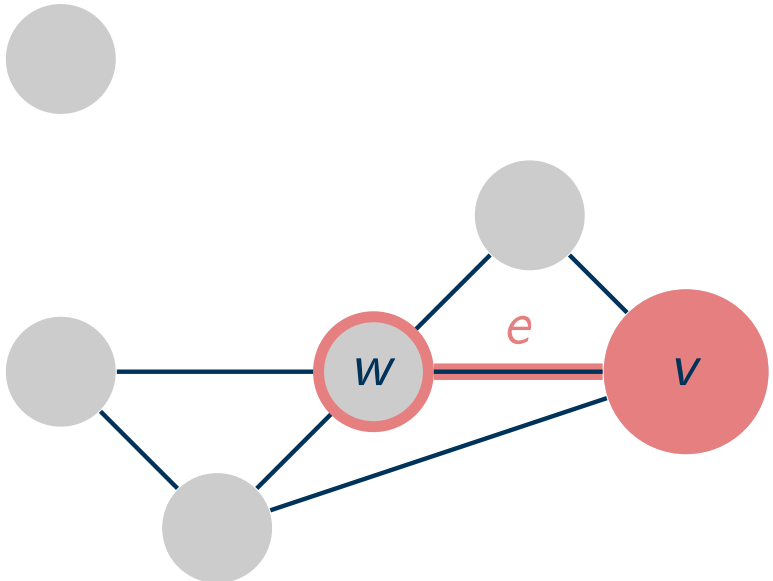

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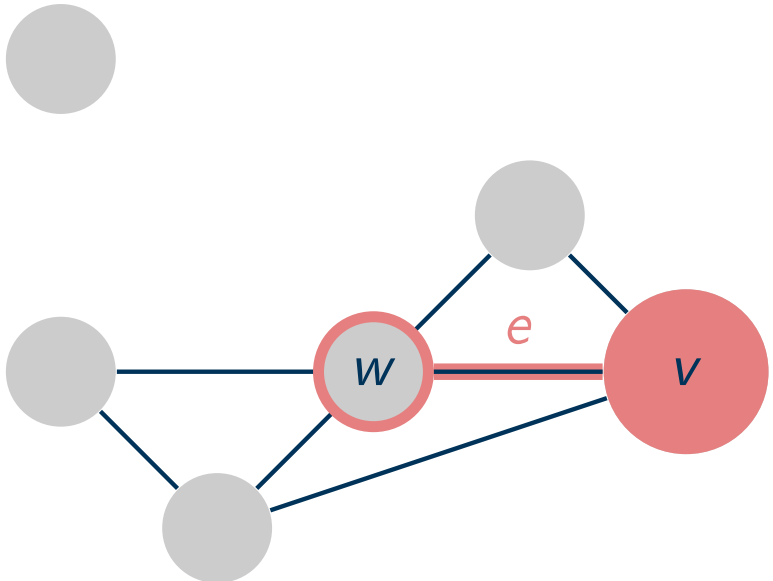

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- if $\{v, w\}$ becomes matching edge, by recursion, replace $\{v, w\}$ with $\{v_1, w\}$ and add $\{u, v_2\}$

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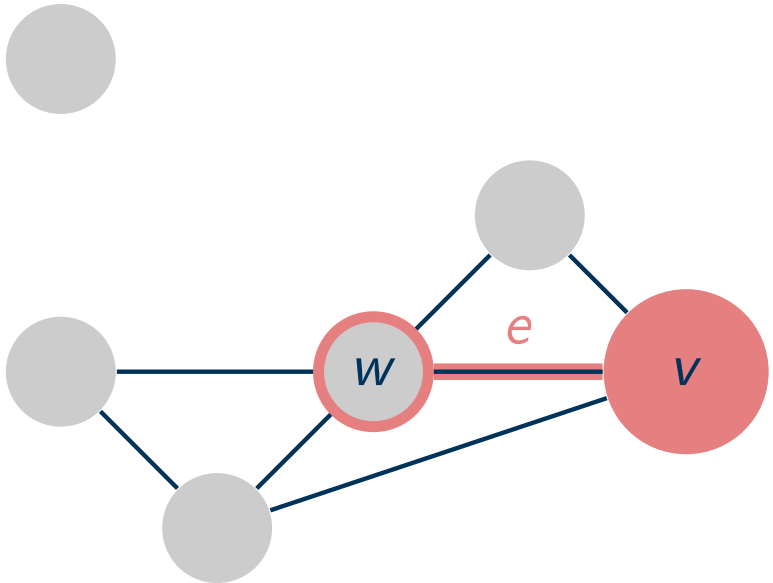

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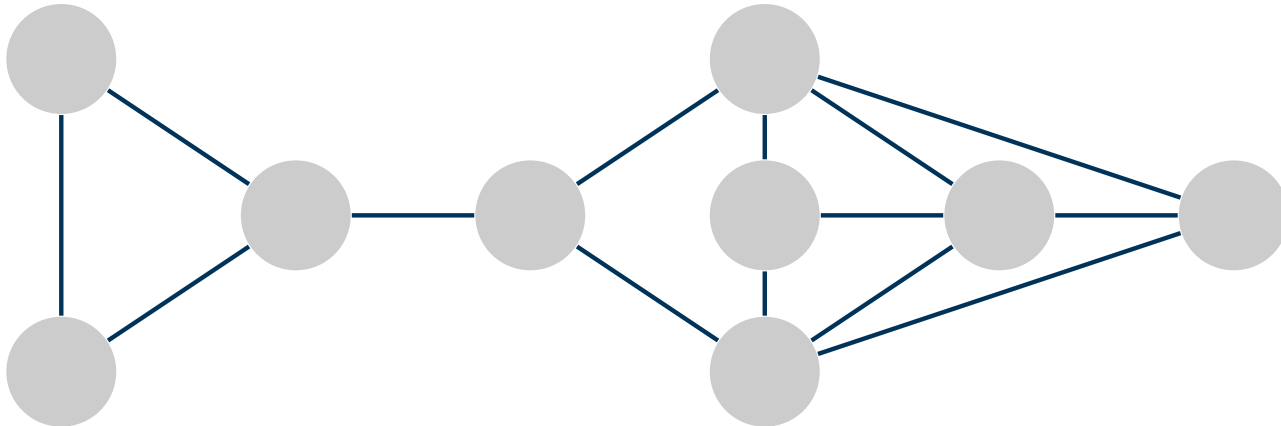
- if $\{v, w\}$ becomes matching edge, by recursion, replace $\{v, w\}$ with $\{v_1, w\}$ and add $\{u, v_2\}$, or replace $\{v, w\}$ with $\{v_2, w\}$ and add $\{u, v_1\}$

Heuristic Reduction Steps



Choose an edge $e = \{u, v\}$ to put in the matching and shrink the graph.

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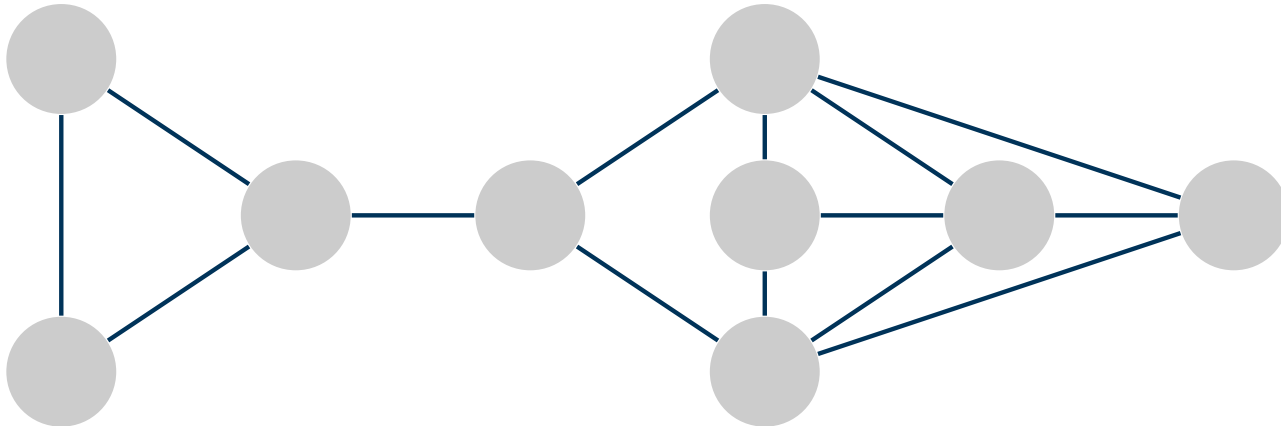

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HEU(rand)

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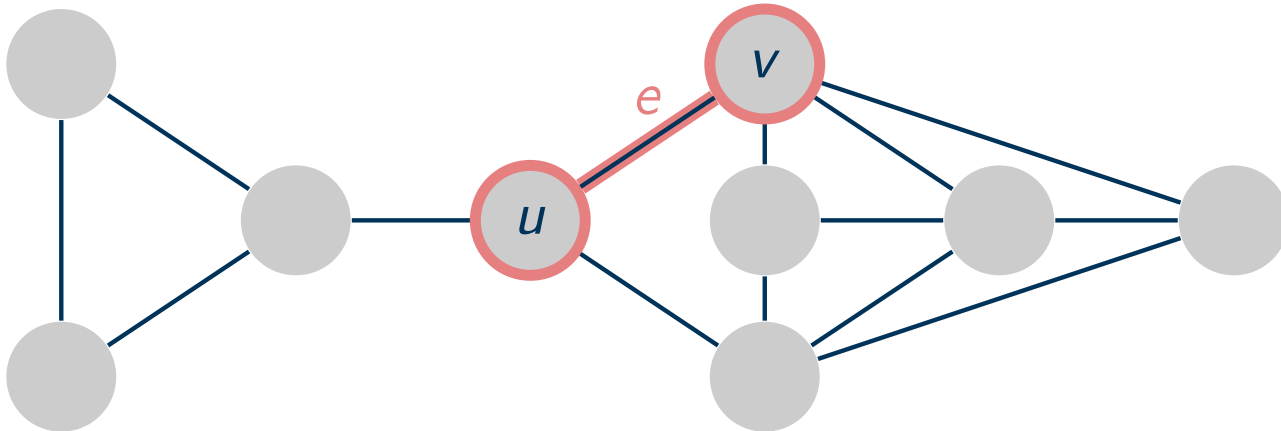

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HEU(rand)
- e is randomly chosen

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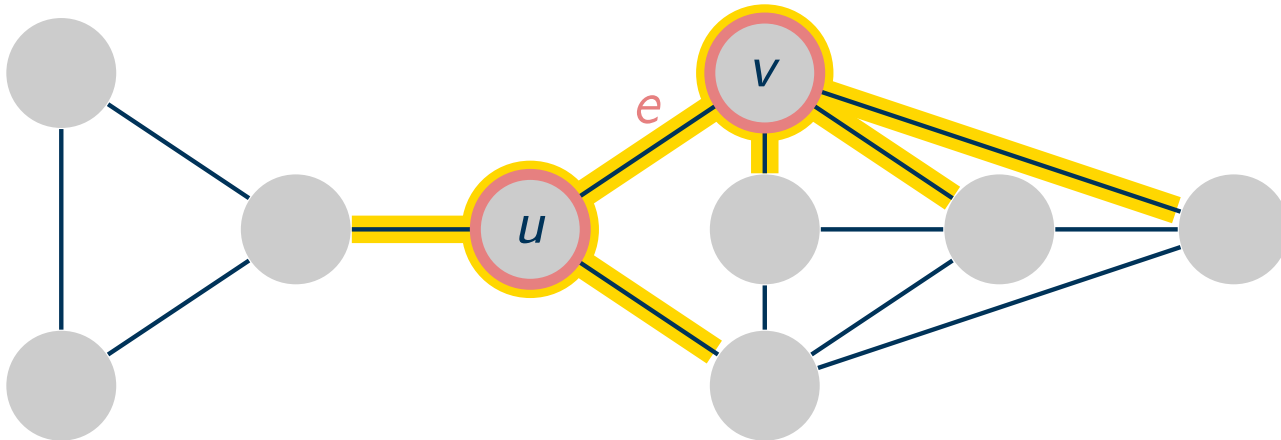

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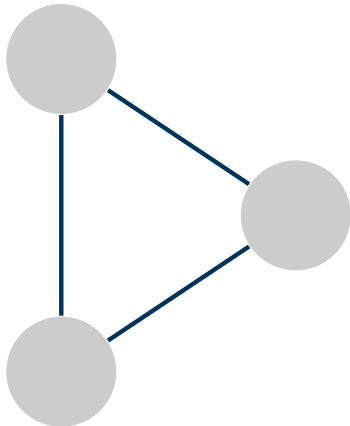

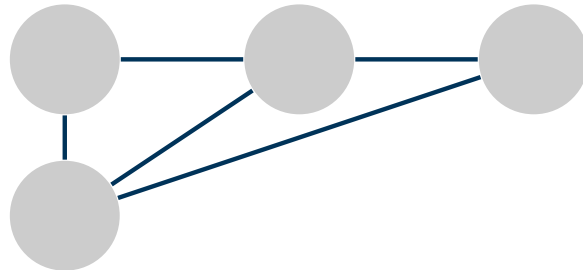

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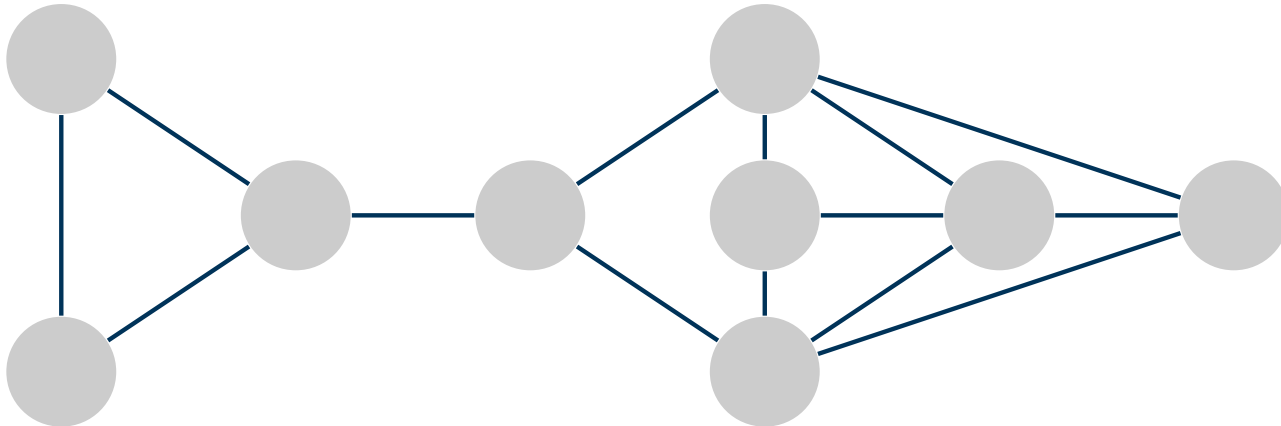

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HEU(deg,deg)

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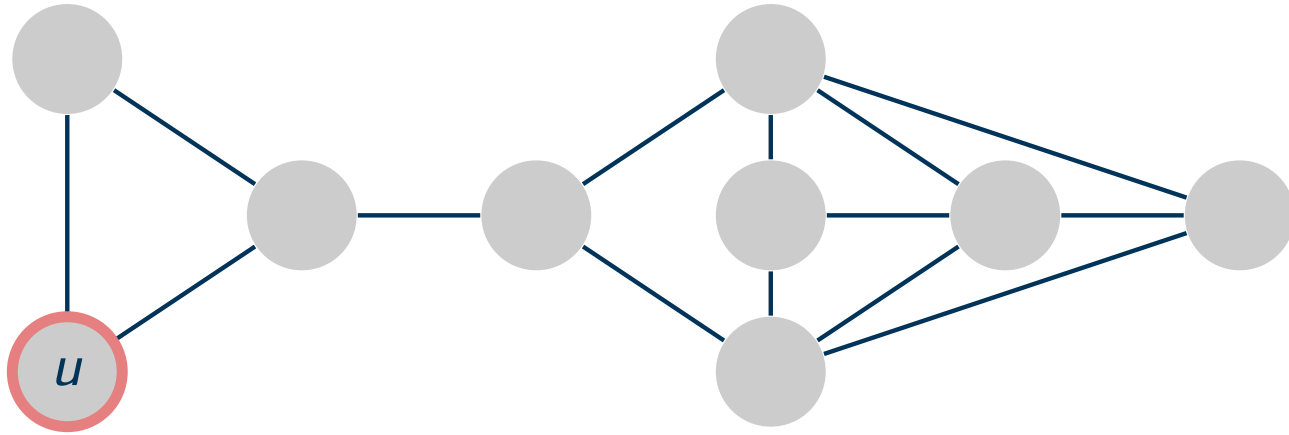

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HEU(deg,deg) ▸ u is a node of smallest degree

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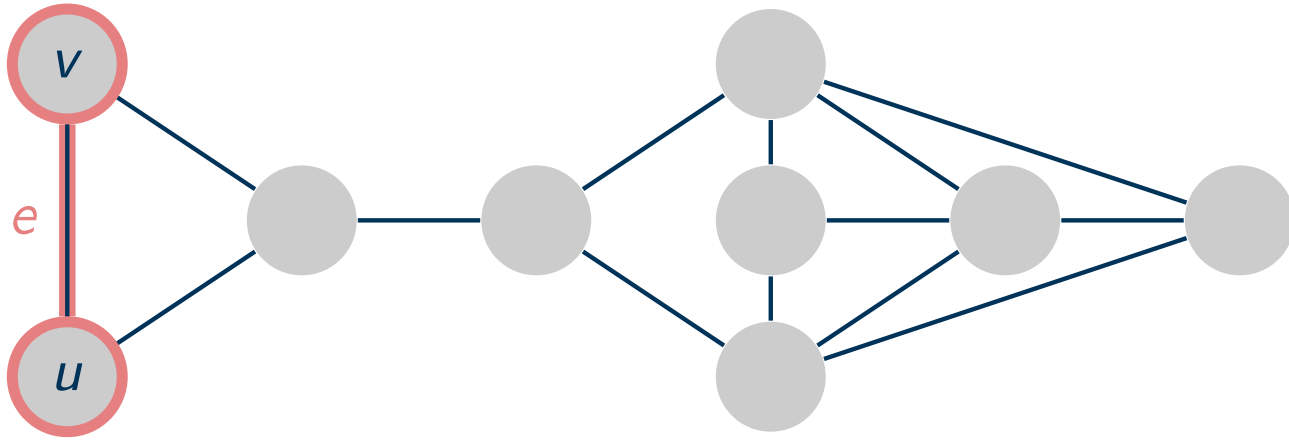

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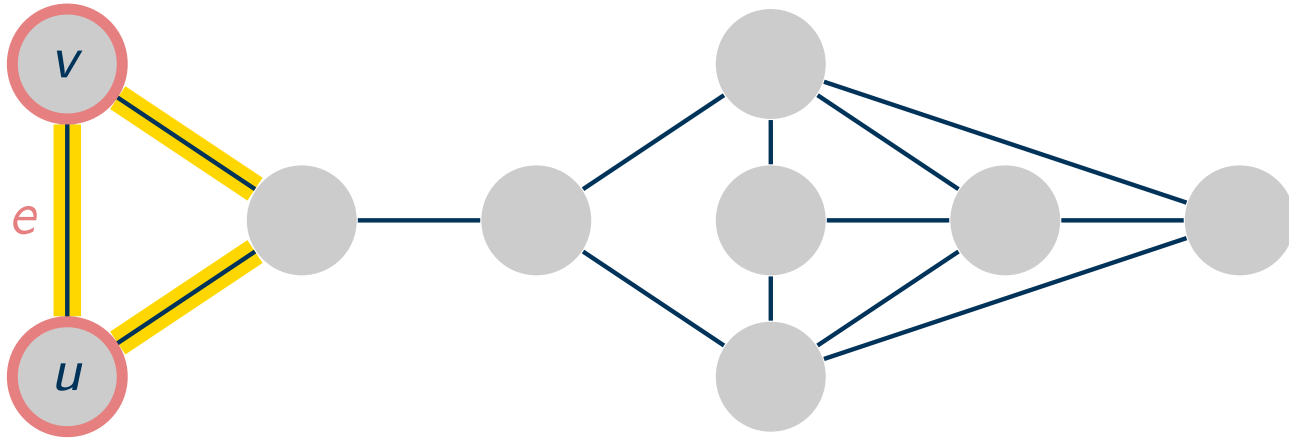

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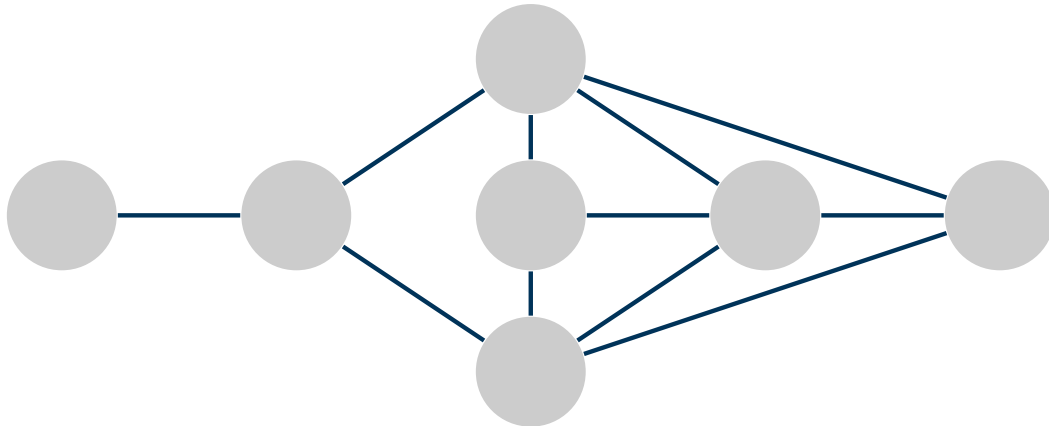

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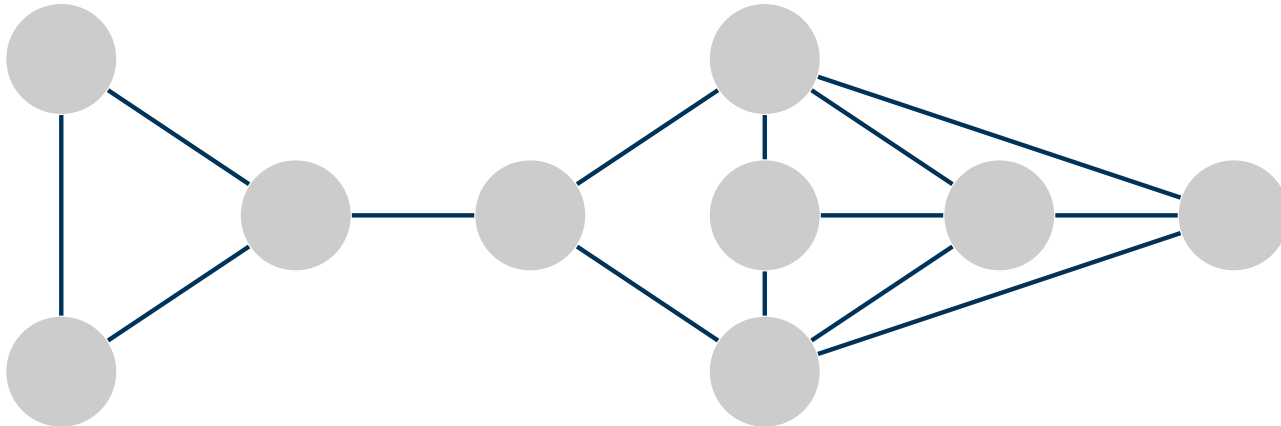

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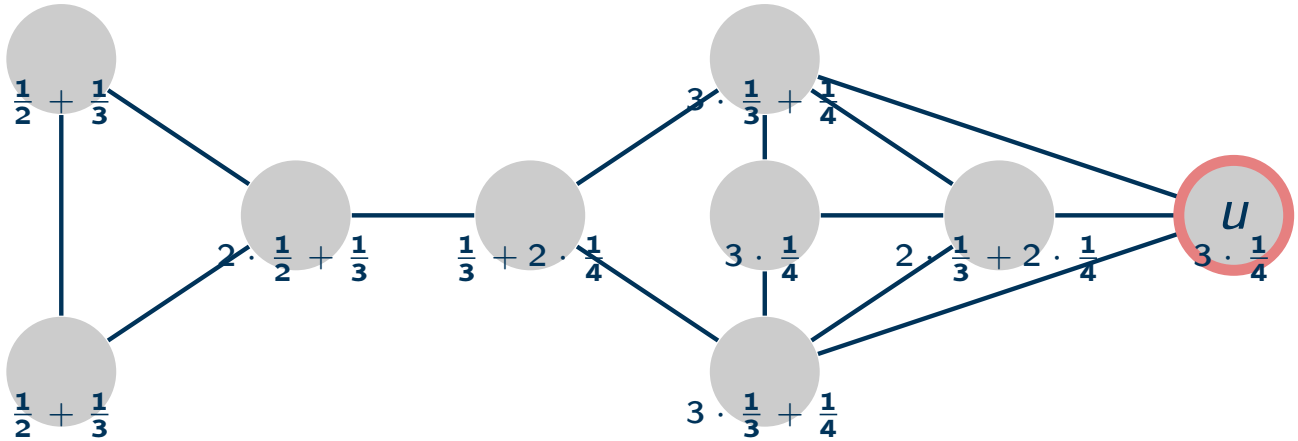

Choose an edge $e = \{u, v\}$ to put in the matching and shrink the graph.

HEU(rand)
- e is randomly chosen

HEU(deg,deg)
- u is a node of smallest degree
- v is a neighbor of u of smallest degree

HEU(pot,deg)
- u is a node of smallest potential $\pi(u)$, where

$$\pi(u) := \sum_{\{u,v\}\in E} \frac{1}{\deg(v)}$$

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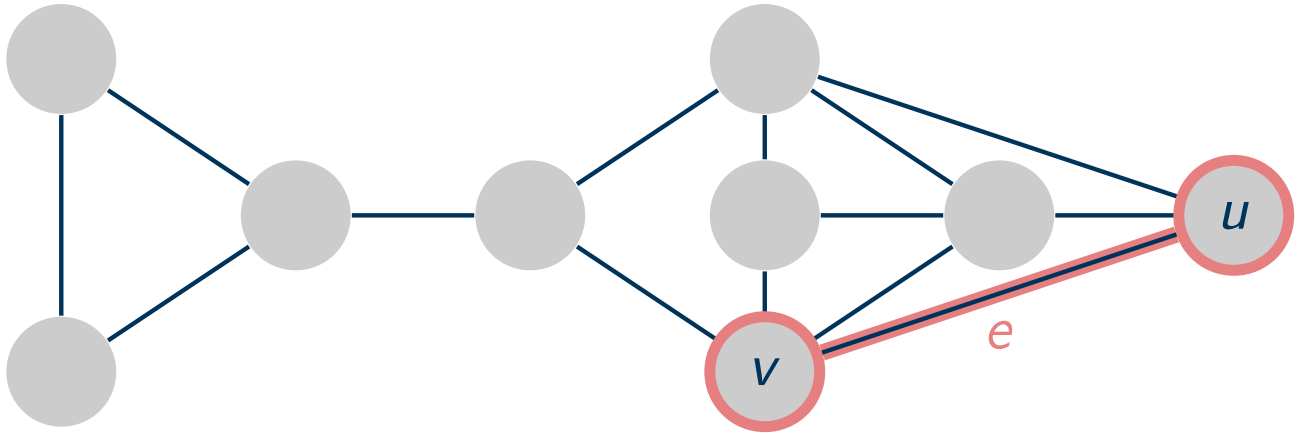

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Greedy Heuristics

"random edge":

- **OPT(1):HEU(rand)** — Karp-Sipser heuristic [Karp and Sipser, 1981]
- **OPT(1,2):HEU(rand)**

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"minimum potential, minimum degree":
new algorithms — adaptation of selfless algorithm by [Sanders, 2004]

- **OPT(1):HEU(pot,deg)**
- **OPT(1,2):HEU(pot,deg)** — heuristic that probably has no critical region

Experimental Setup

graphs:

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- random graphs $G(n; c)$ with n nodes and edge probability $p = c/(n-1)$
- $n = 10^6$, $c \in [1, 10]$, step size 0.1

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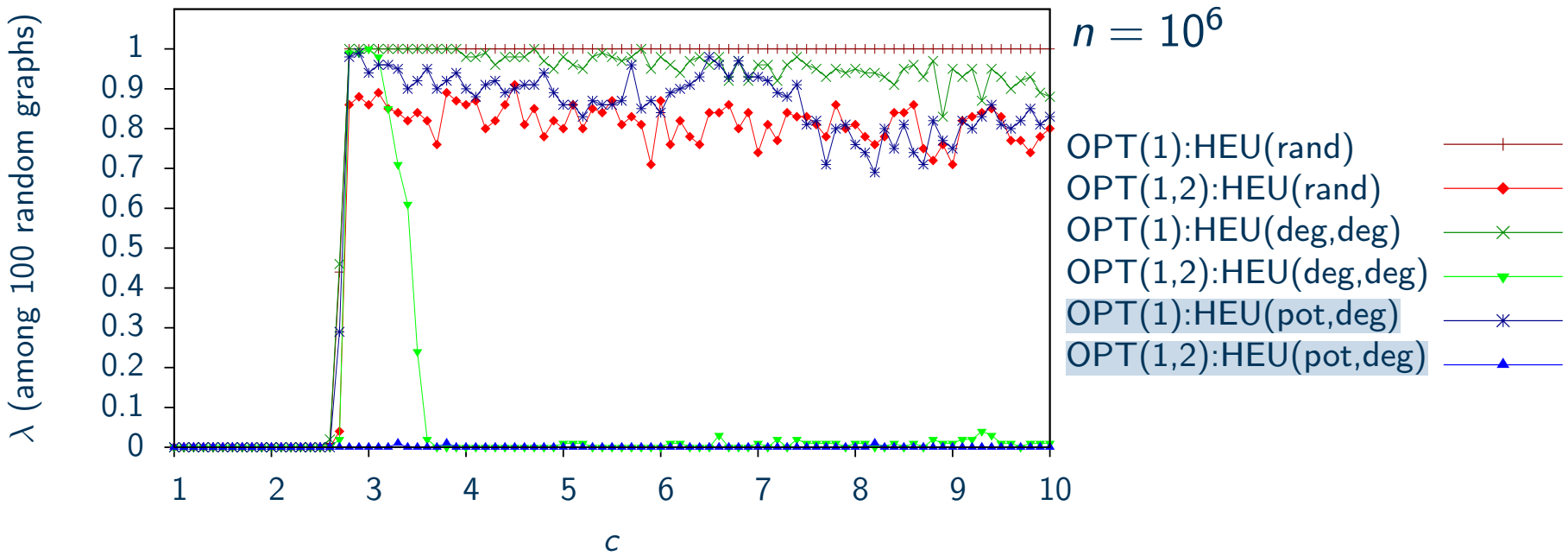
$\overline{\#o1}$: avg. fraction of **OPT(1)** steps

$\overline{\#o2}$: avg. fraction of **OPT(2)** steps

$\overline{\#h}$: avg. fraction of **HEU($\star$)** steps

Failure Rate



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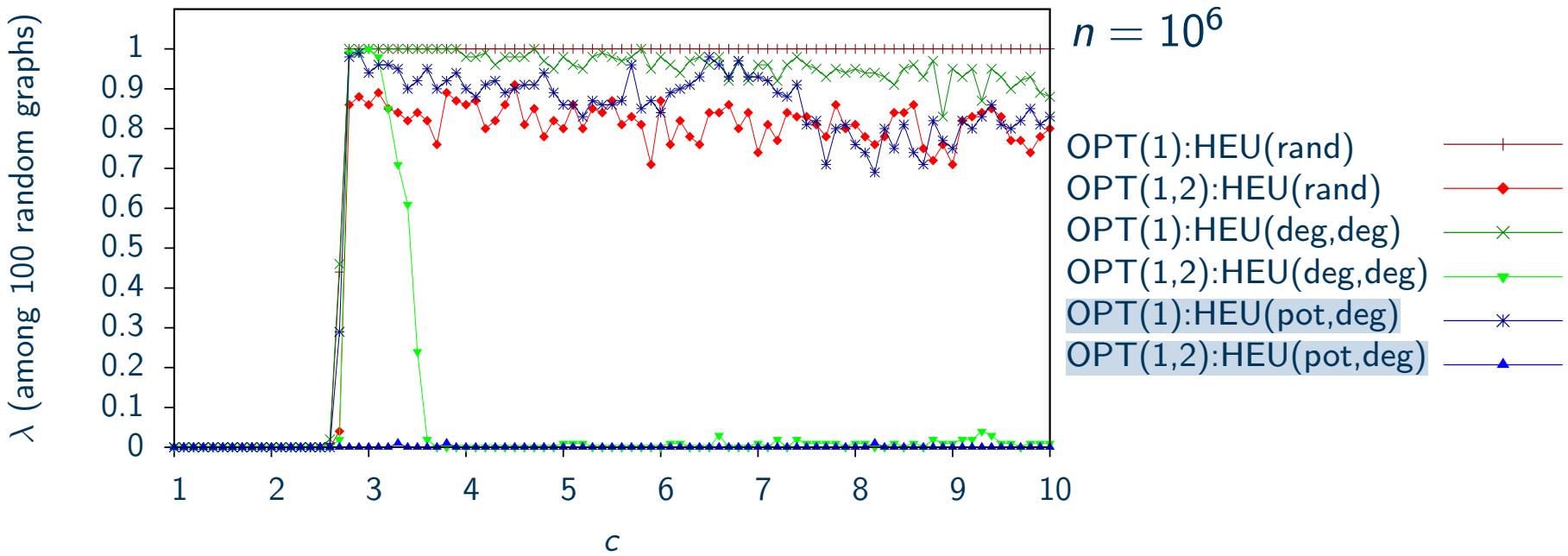

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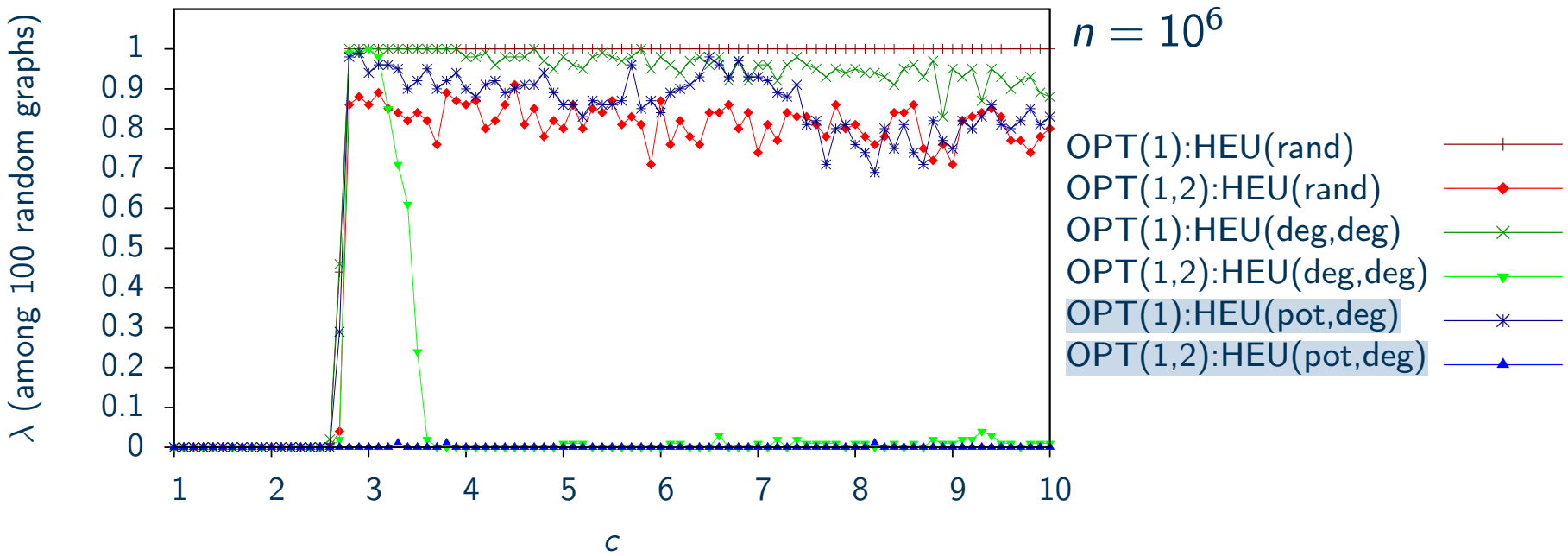

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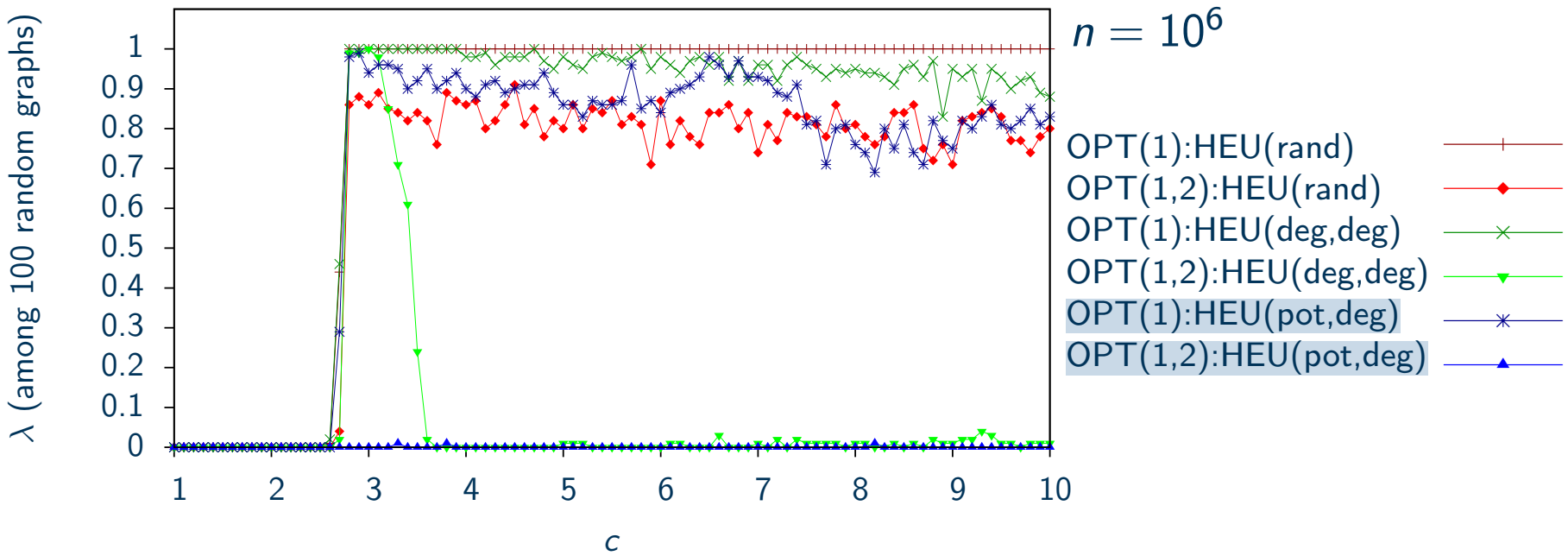

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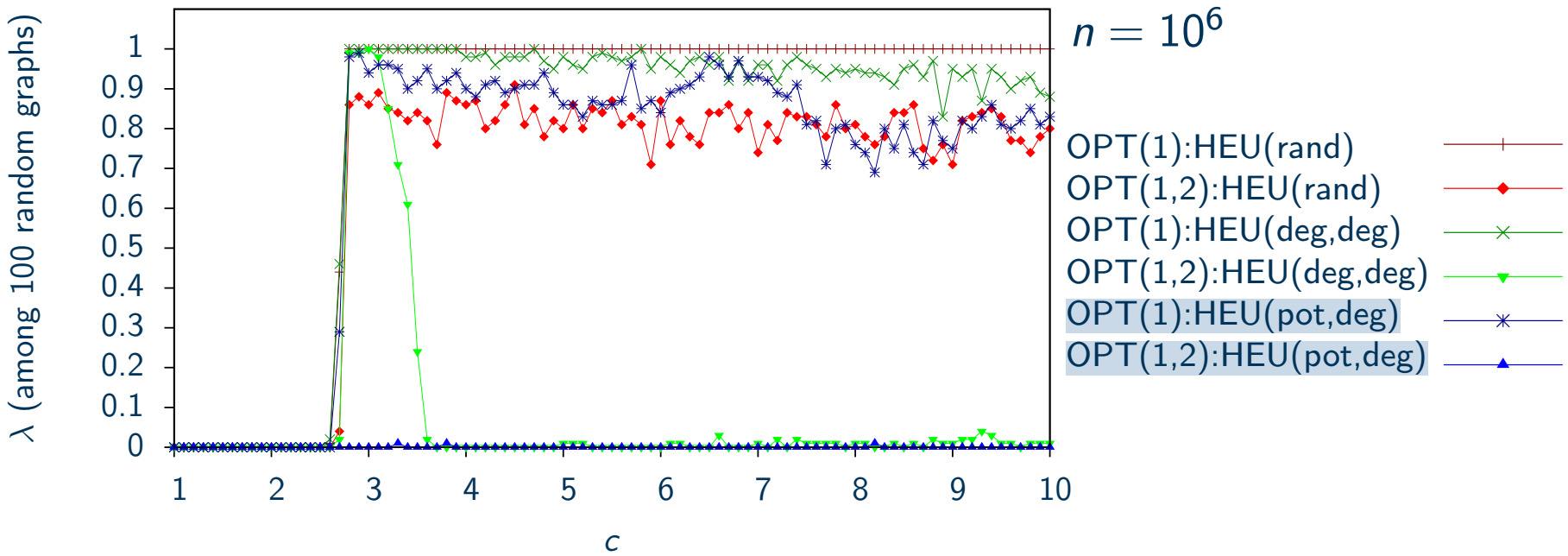

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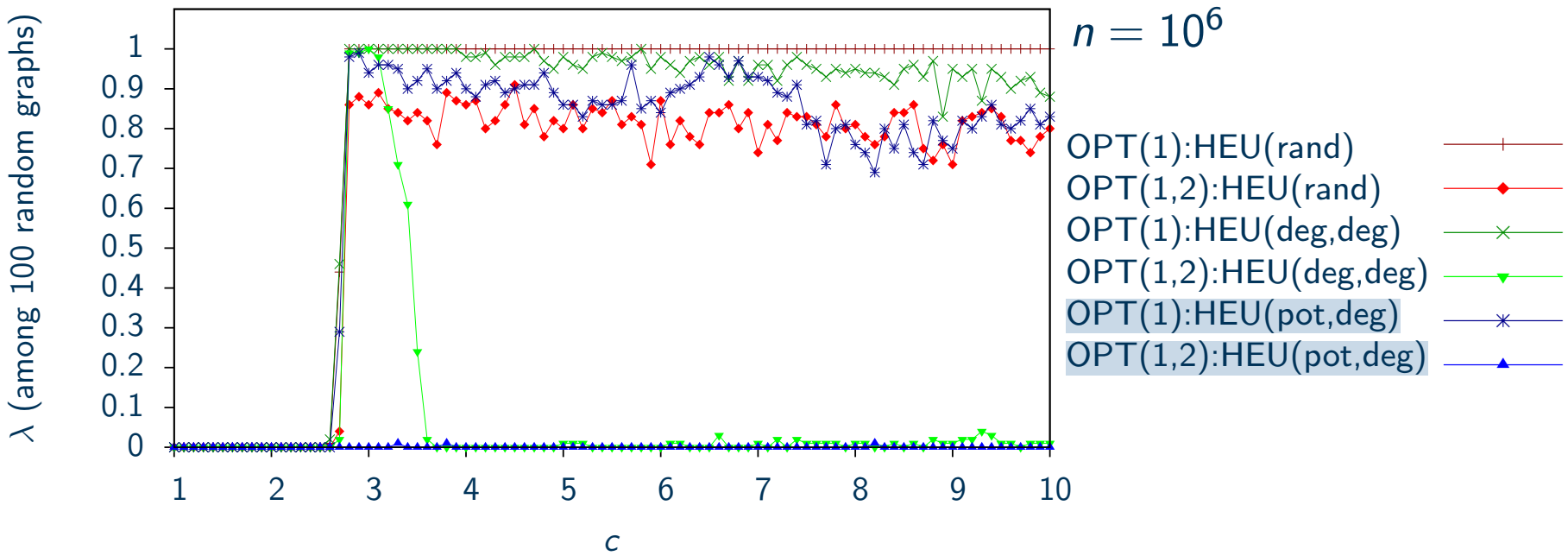

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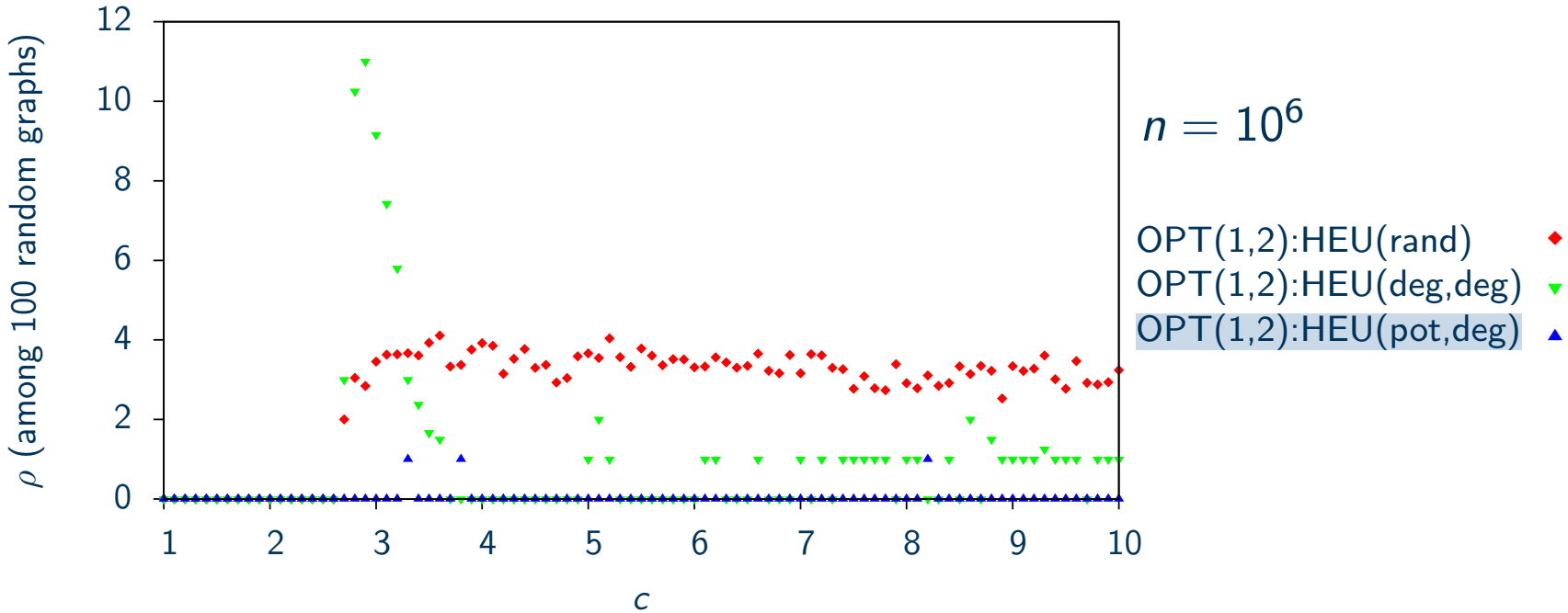
OPT(1,2):HEU(pot,deg) fails only 3 times out of 9100.

Number of Lost Edges



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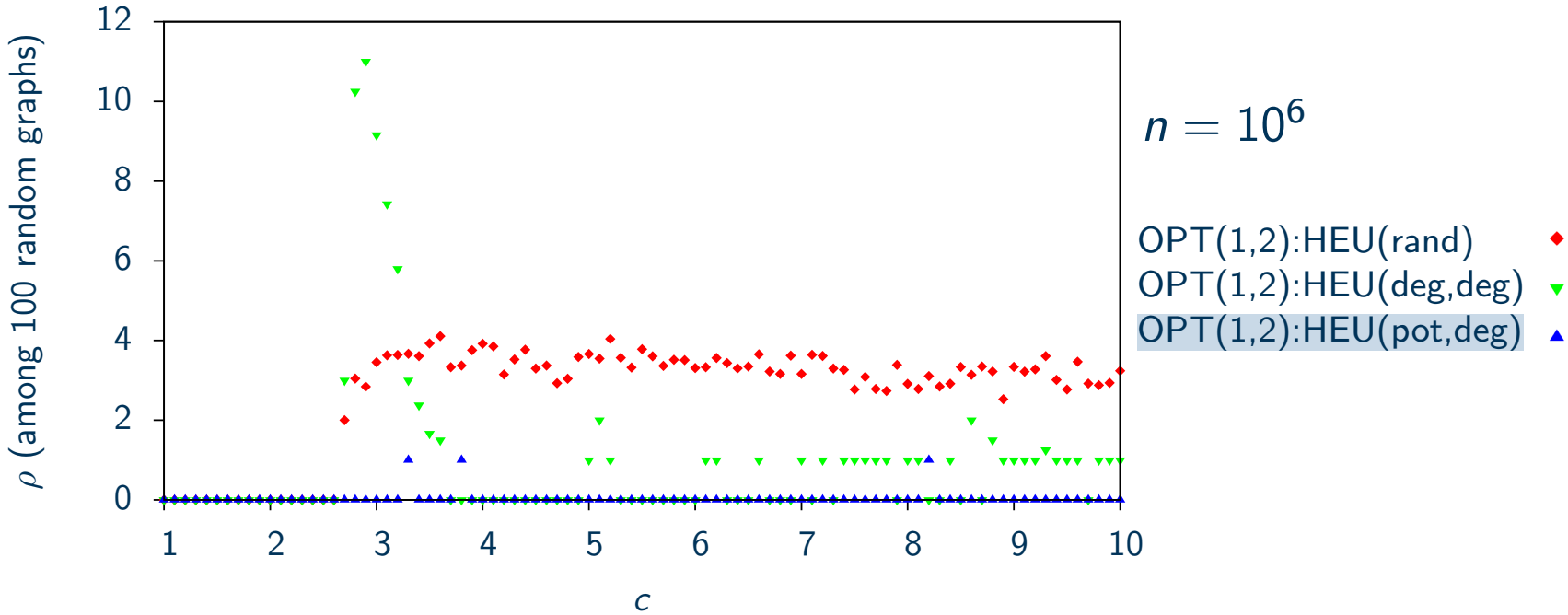

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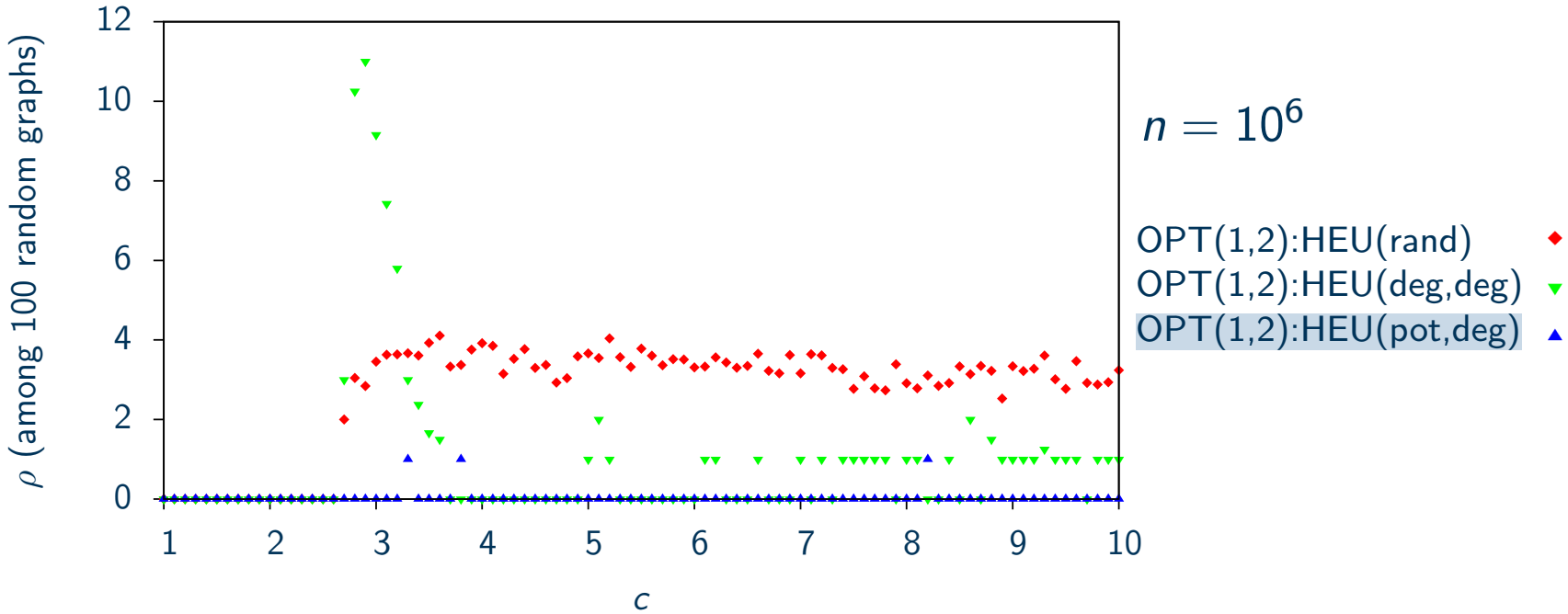

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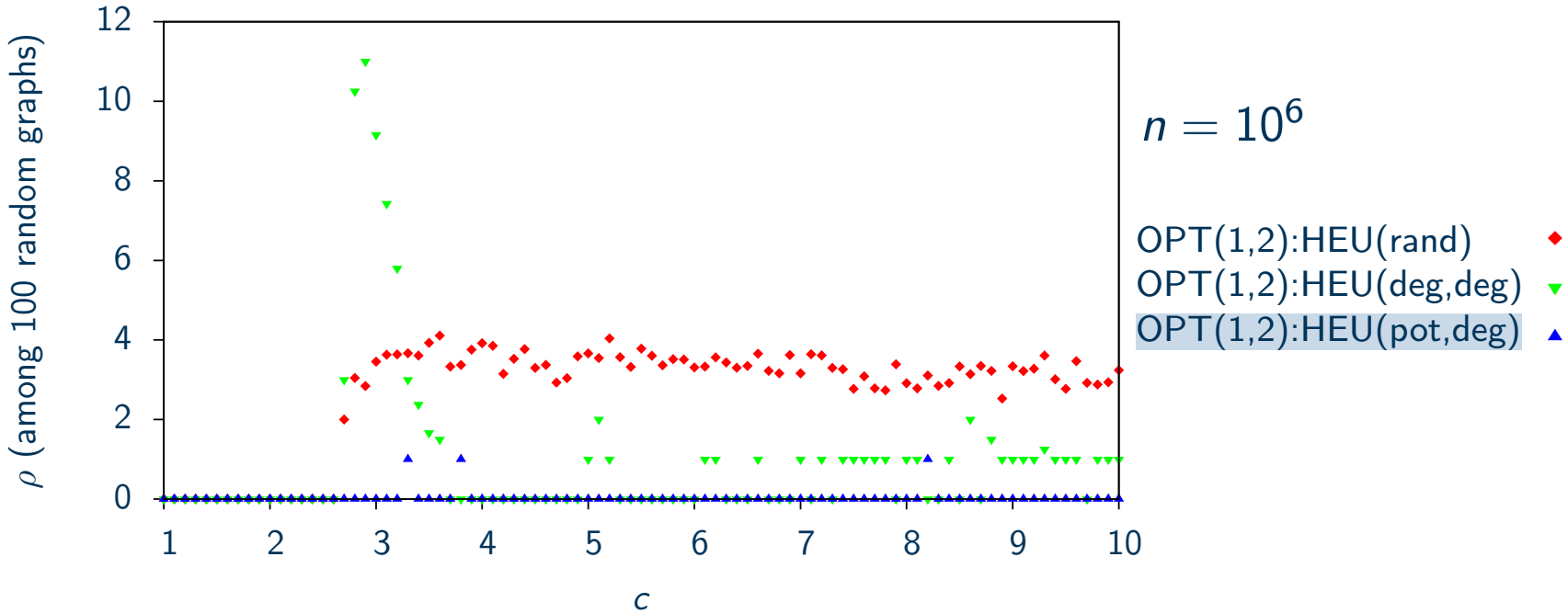

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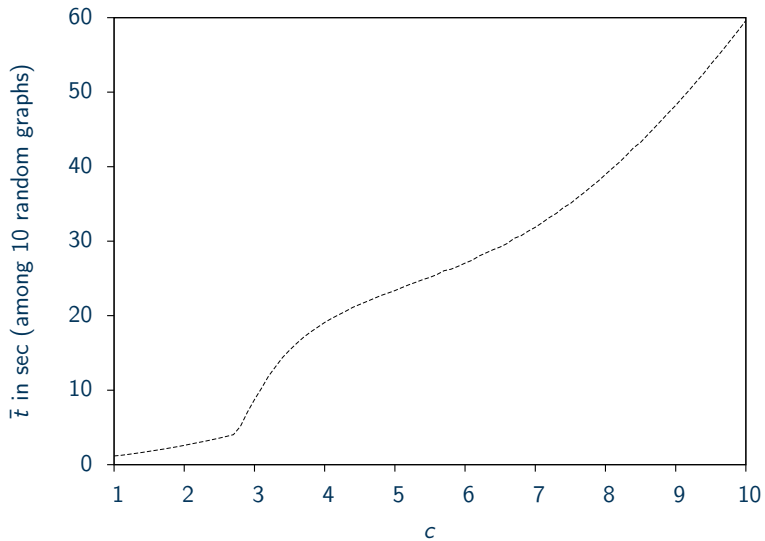
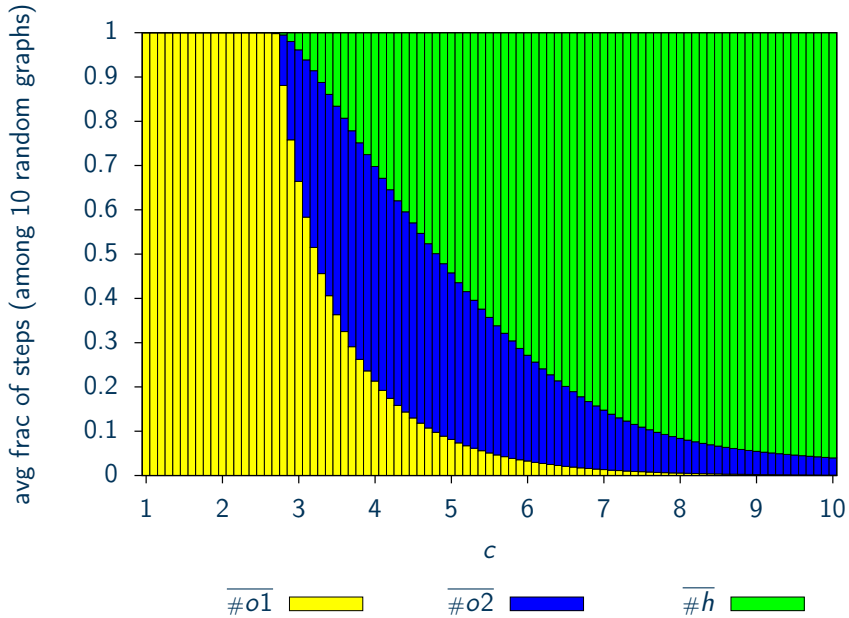
Running Time Behavior

$n = 10^6$

OPT(1,2):HEU(pot,deg)





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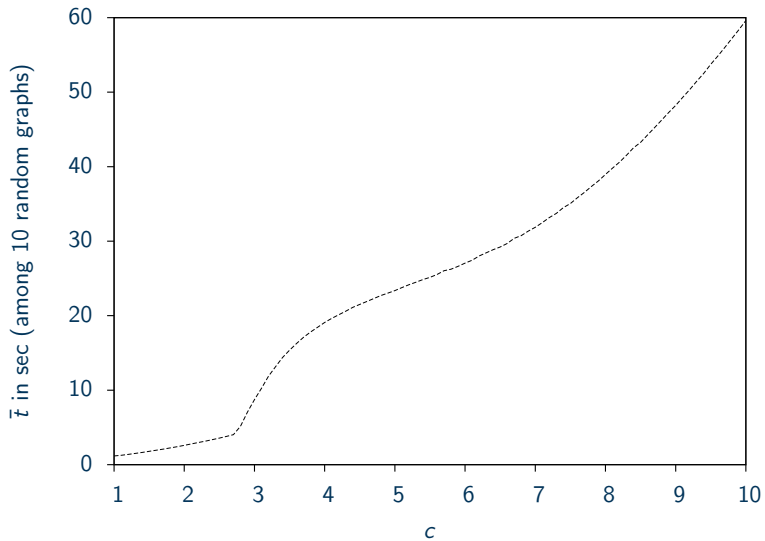

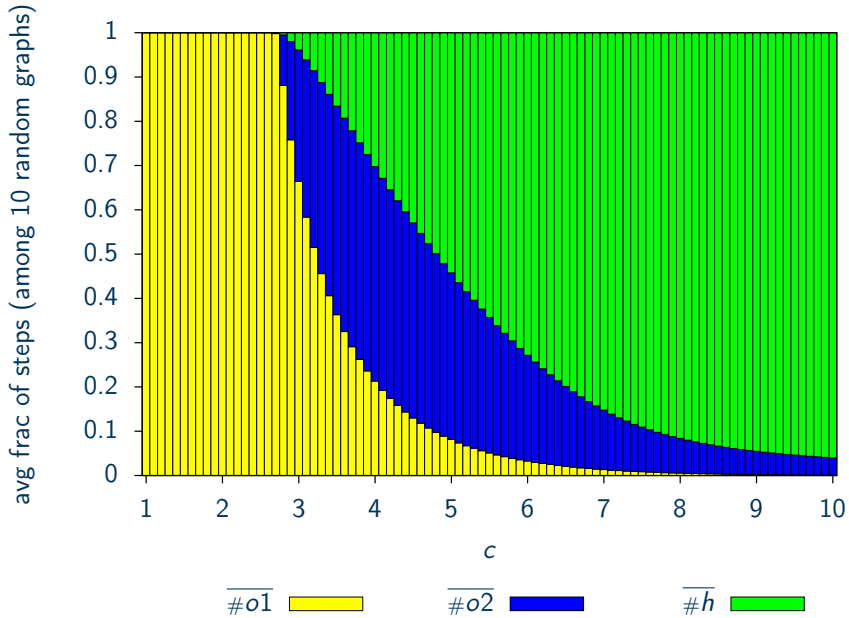

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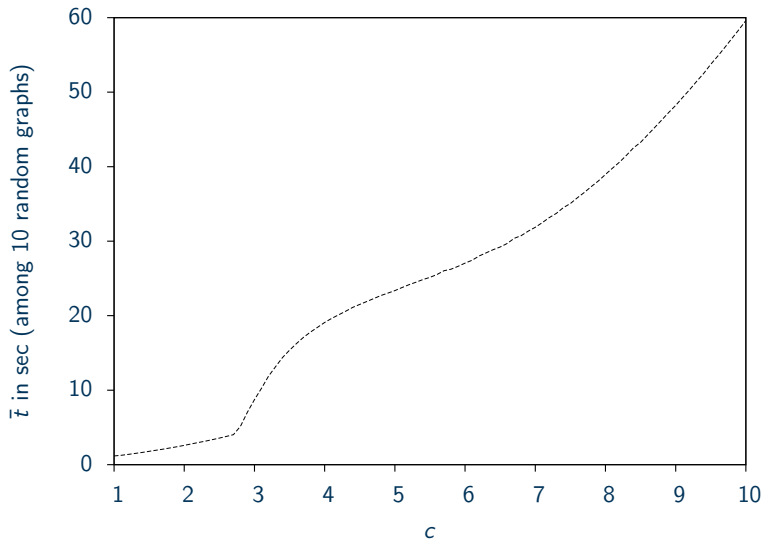

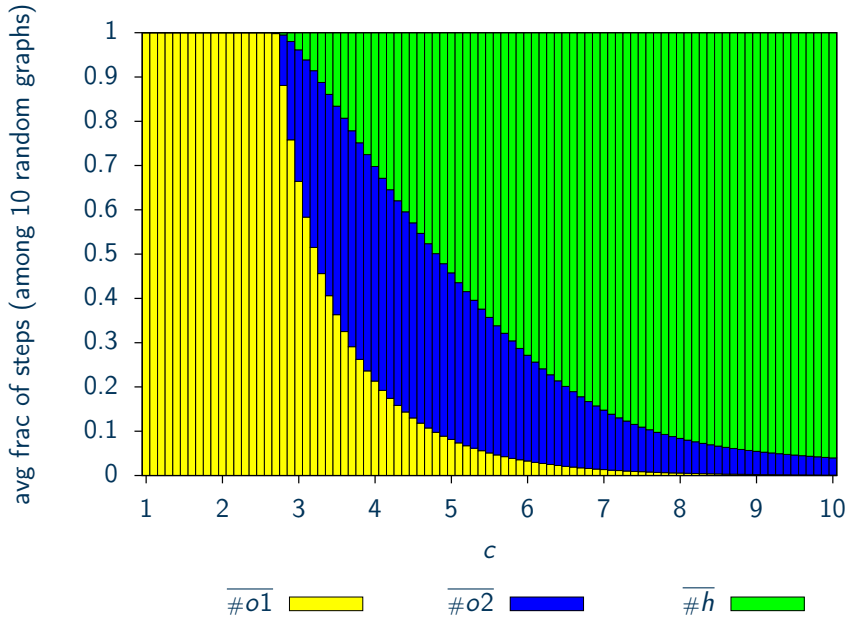

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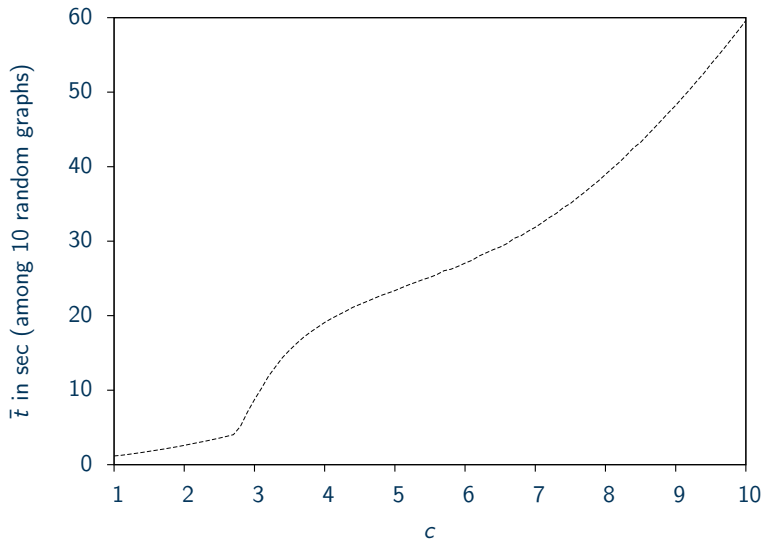

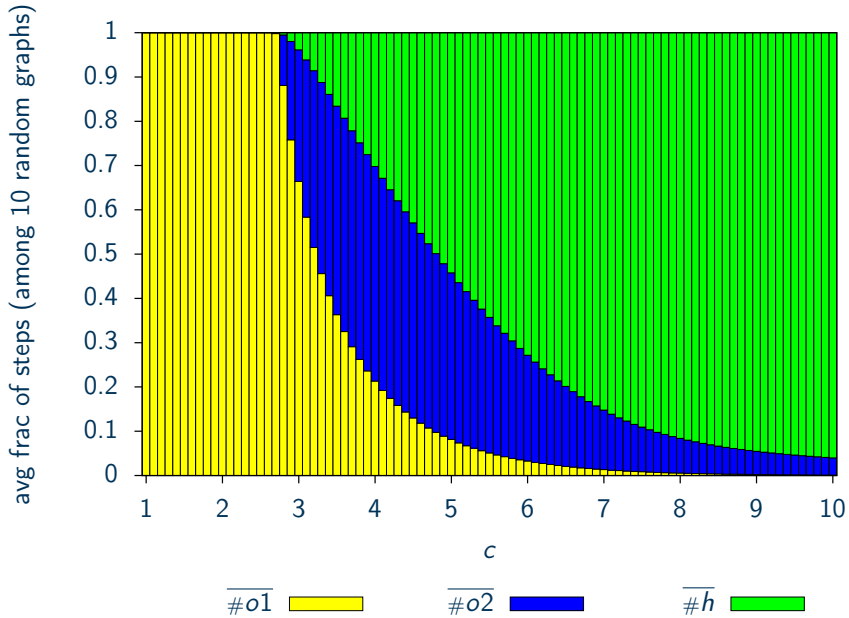

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- $2.7 < c < 6$: non-linear slope — strong decrease of $\overline{\#o1}$, non-linear increase of $\overline{\#o2}$ and linear increase of $\overline{\#h}$
- $c \geq 6$: slightly non-linear slope — dominated by $\overline{\#h}$

Running Time Behavior

exemplary comparison with exact algorithm:

$\bar{t}$ in seconds (average among 10 random graphs), $n = 10^6$

c	$\bar{t}(H)$	$\bar{t}(E)$
1		
5		
9		

H: **OPT(1,2):HEU(pot,deg)**

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c	$\bar{t}(H)$	$\bar{t}(E)$	$\bar{t}(E+i)$
1	1.1	73.1	0.8
5	23.5	948.6	6.6
9	48.4	1216.5	8.3

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OPT(2) and **HEU(pot,deg)** steps are slow!
(At least in our implementation.)

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- Apply "selfless approach" to other (harder) problems, like graph coloring.

Thank you!

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