SEA 2012 – Bordeaux

Implementation and Comparison of Heuristics for the Vertex Cover Problem on Huge Graphs*

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*Work supported by the project ANR-09-EMER-010 *TODO*

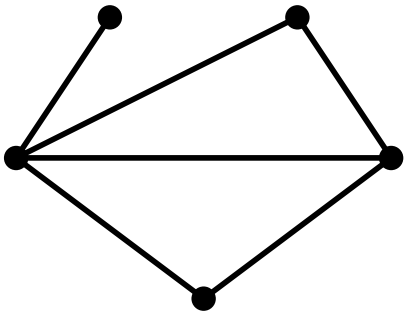
The Vertex Cover problem...

Definition (Cover)

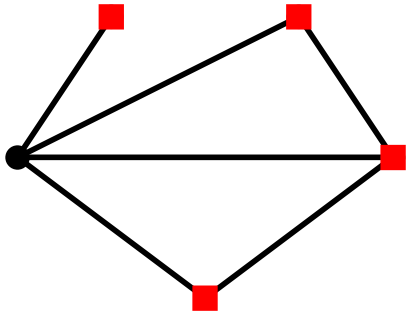
Let $G = (V, E)$ be any simple graph. C is a cover *if $C \subseteq V$ and $\forall e = uv \in E$, $u \in C$ or $v \in C$ (or both).*

Find a *cover* of minimum size

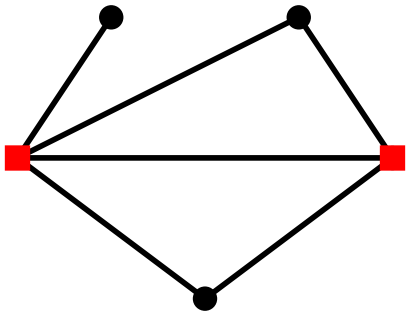
$\rightarrow$ well-known **NP**-complete optimization graph problem



A graph



A cover



An optimal cover

... on huge graphs

It appears in various applied problems:

- network monitoring;
- SNP haplotype assembly problem *(DNA sequence alignment)*;
- etc.

where data sizes can be very important.

> *"The amount of data doubles every 20 months!"*

Previous works

Theoretical study:

→ *Analysis and Comparison of Three Algorithms for the Vertex Cover Problem on Large Graphs with Low Memory Capacities*, Algo. Op. Res., Vol. 6, 2011

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Experiments for the Vertex Cover problem:

- sparse graphs with less than 10,000 vertices

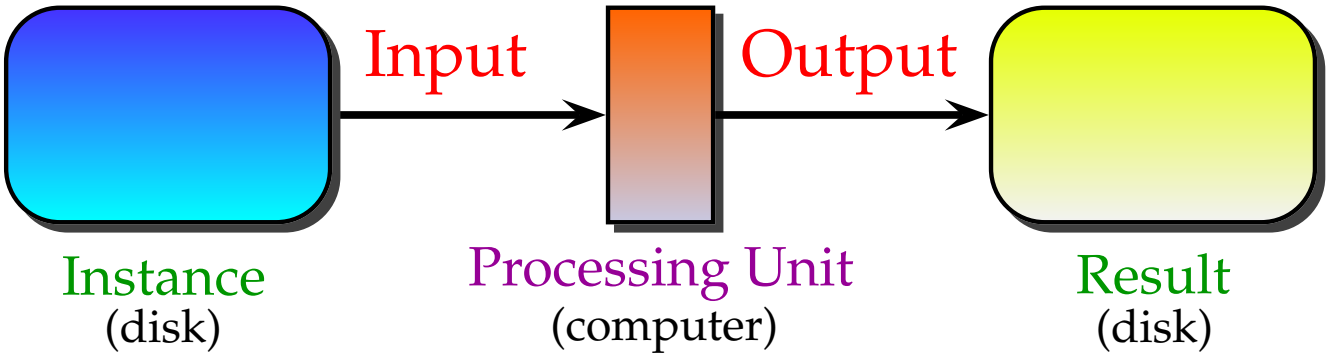
Max sizes $\approx 100 \cdot 10^9$ in our experiments!

Outline

Outline

Implementation

Experimental model



- **Instance:** stored on a external hard disk (*read only* access)
- **Processing unit:** a standard computer (*a laptop!*)

→ low memory capacities

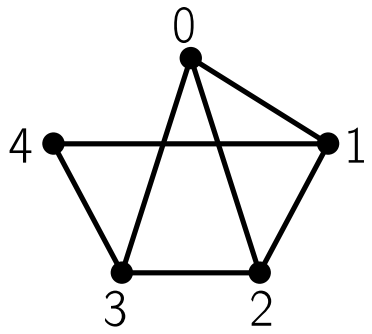
- **Result:** written *as soon as it is produced* on the external disk

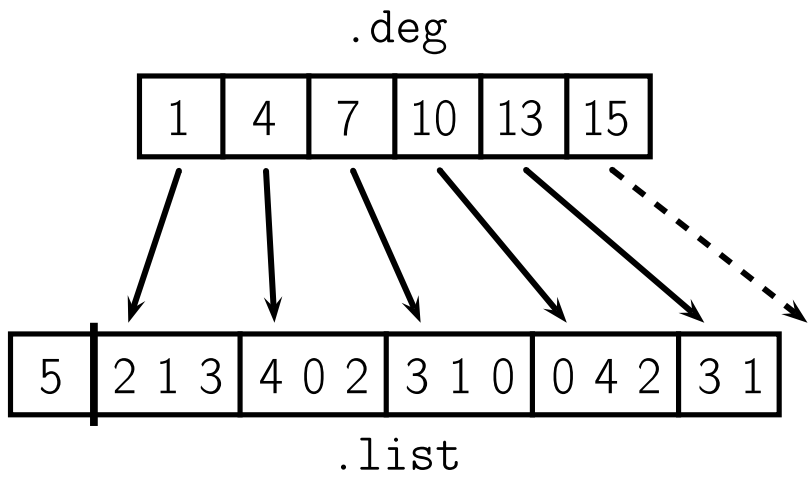
Storage and reading of graphs

With two files

`.list`: list of neighbors of vertices

`.deg`: *pointers* delimiting the neighborhood of each vertex





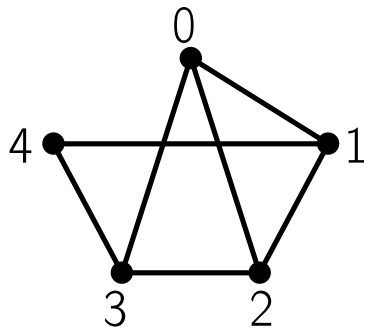
- reading of `.list` file helped by the `.deg` file

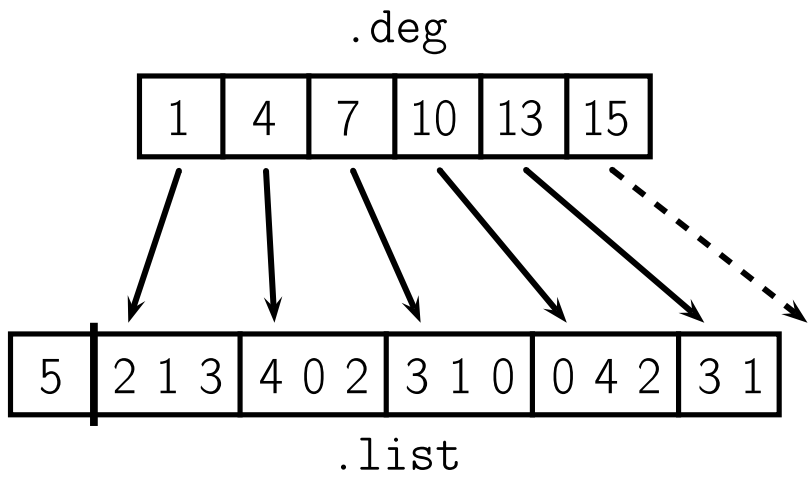
Storage and reading of graphs

With two files

`.list`: list of neighbors of vertices (value n at the beginning) $\rightarrow 2m + 1$ values

`.deg`: *pointers* delimiting the neighborhood of each vertex (degrees computing) $\rightarrow n + 1$ values





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Description of the six implemented algorithms

Let $G = (V, E)$ be a graph C the cover under construction.

For each vertex $u \in V$,

LR: if $u \notin C$, then $\{v \mid uv \in E \wedge v \notin C\}$ is put in C;

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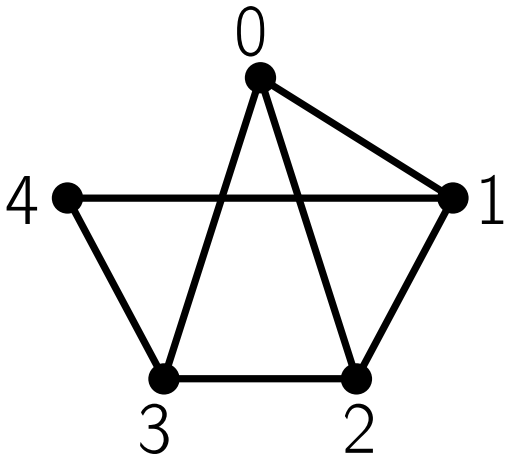
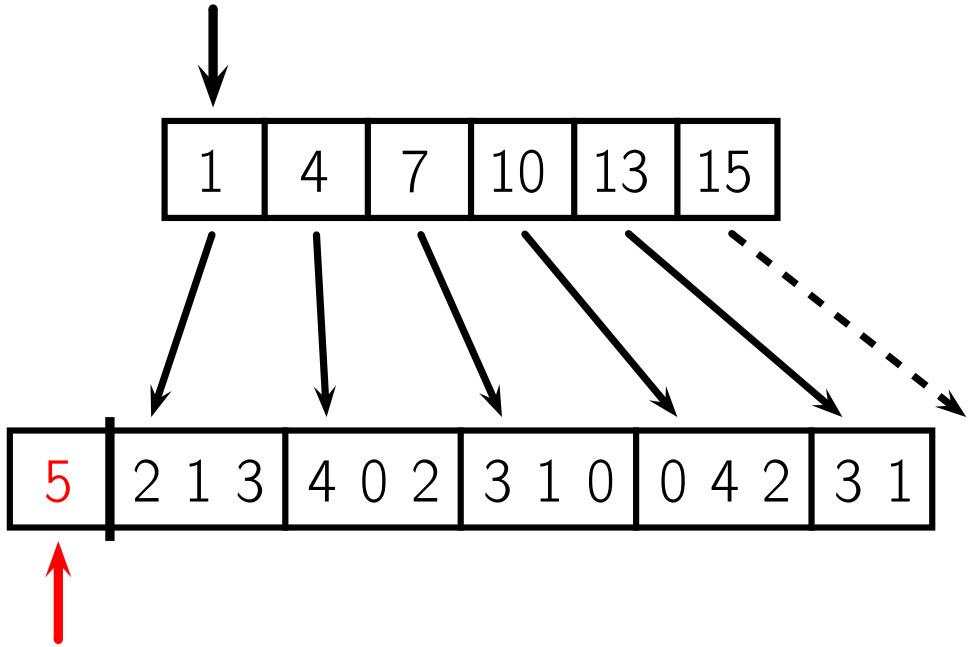
LL: u is put in C iff $\exists v \in N(u)$ such that $v > u$;

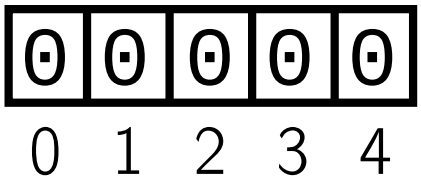
SLL: u is put in C iff $\exists v \in N(u)$ such that $d(v) < d(u)$ or $d(v) = d(u)$ and $v > u$;

ASLL: u is put in C iff $\exists v \in N(u)$ such that $d(v) > d(u)$ or $d(v) = d(u)$ and $v < u$.

Example of execution of algorithm LR







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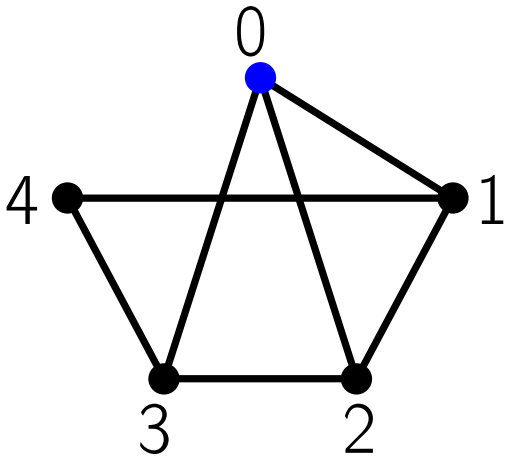

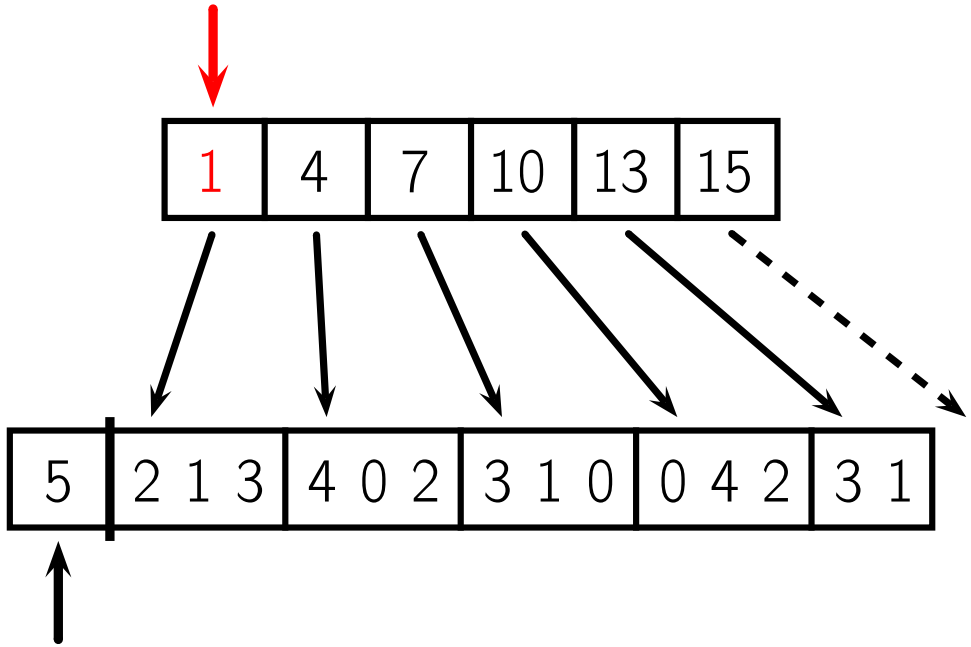

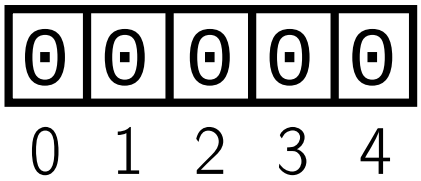

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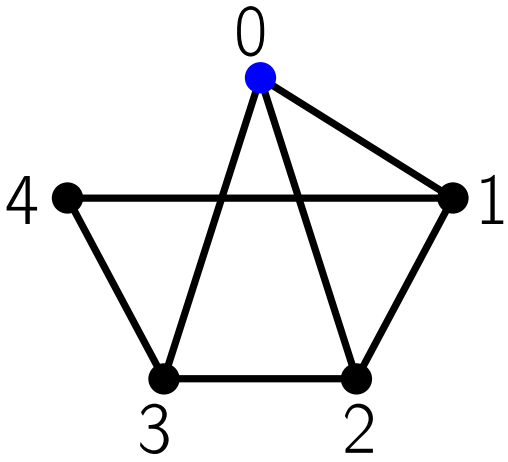

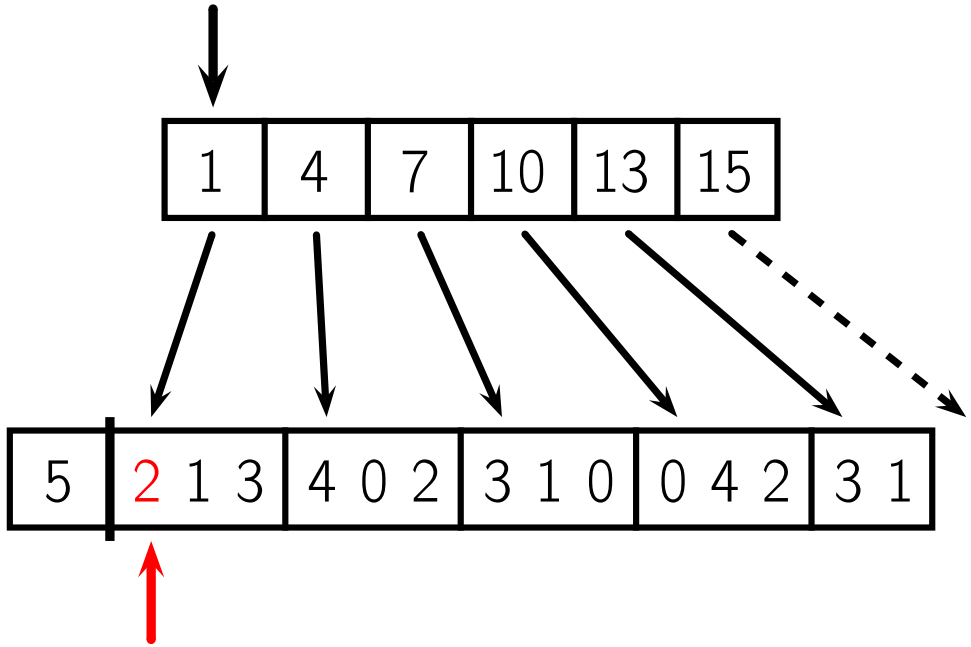

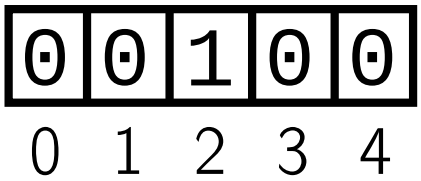

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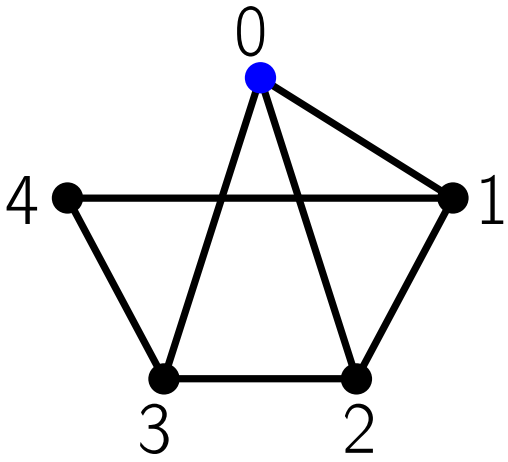

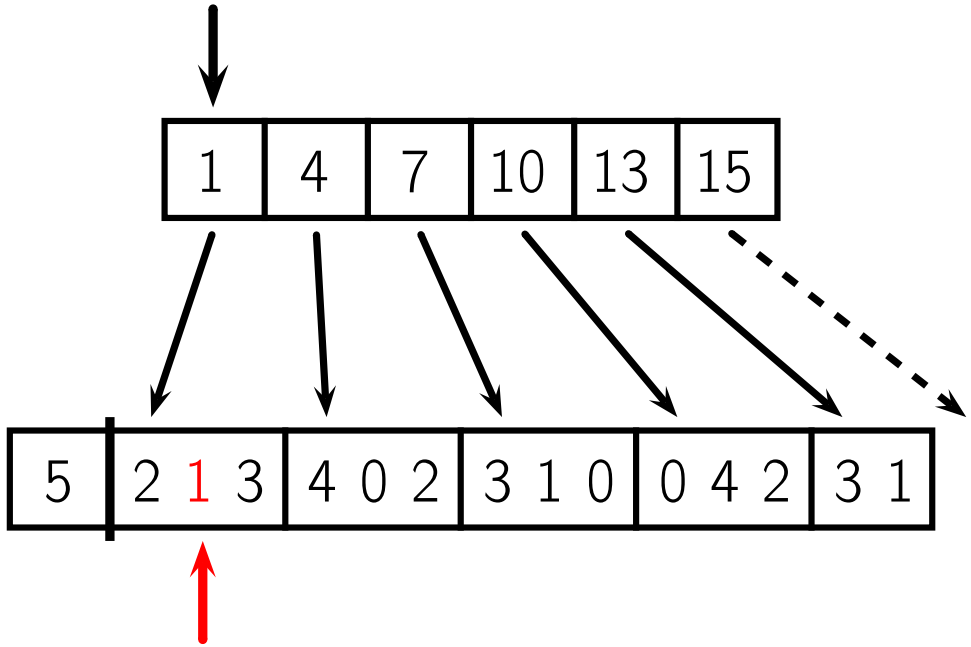

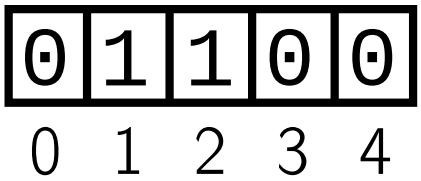

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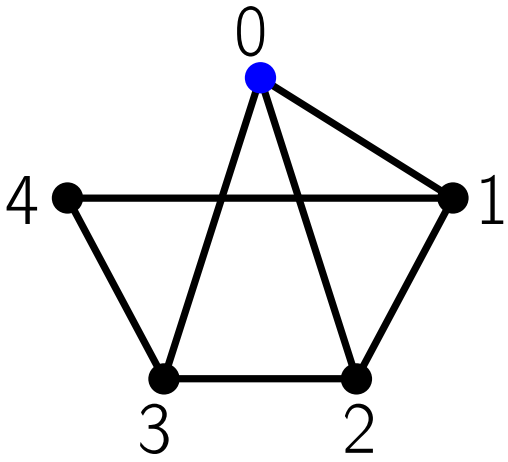

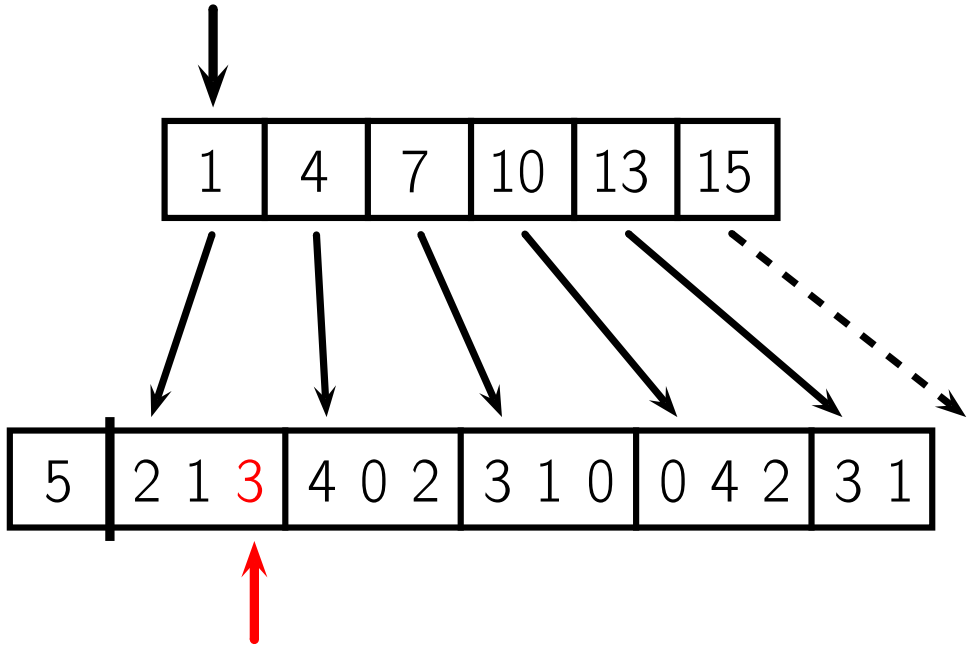

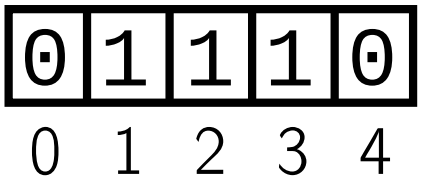

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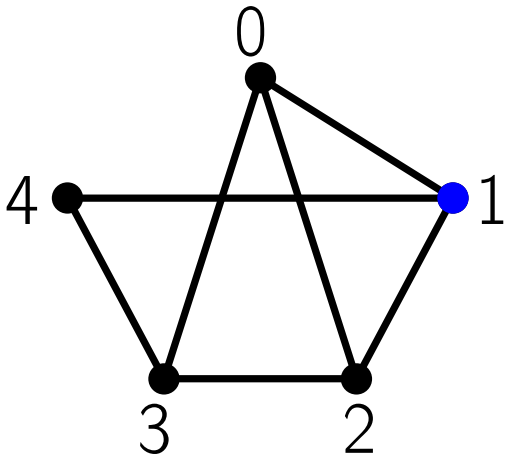

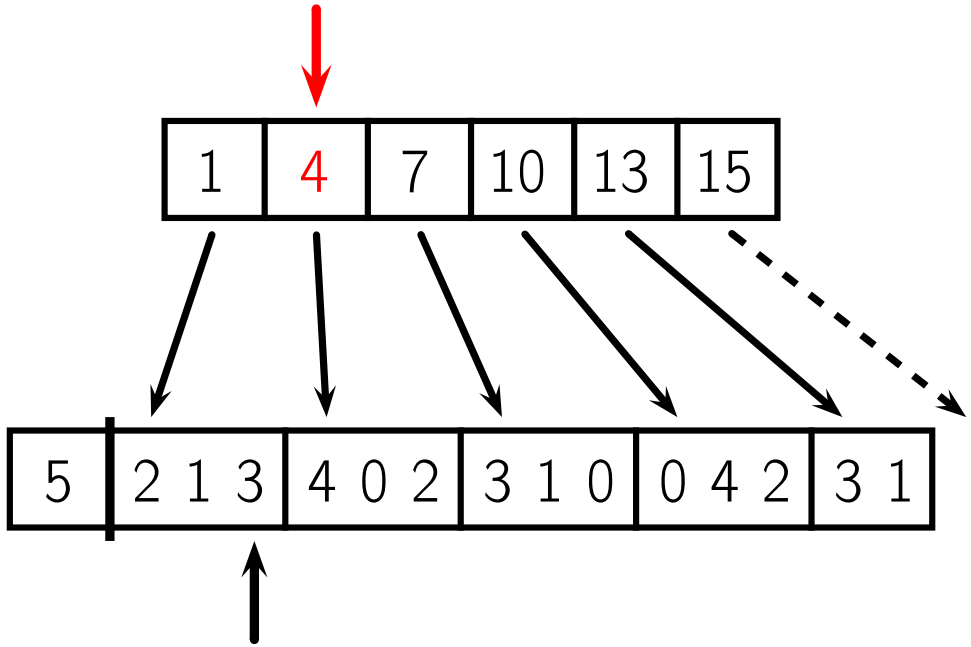

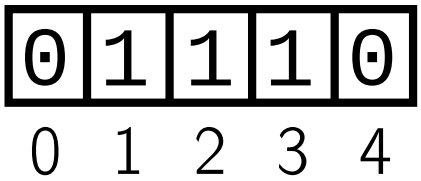

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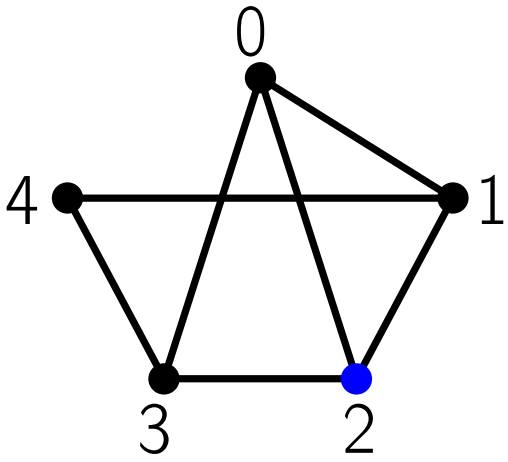

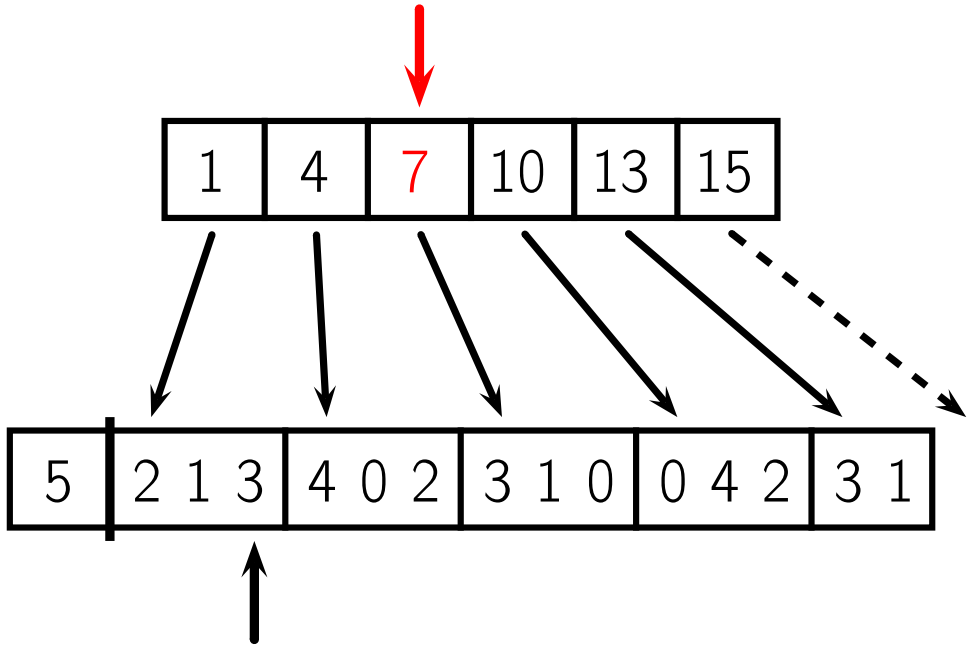

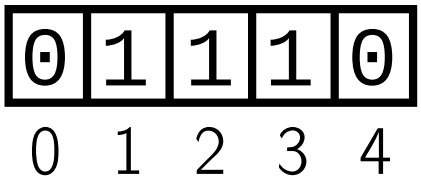

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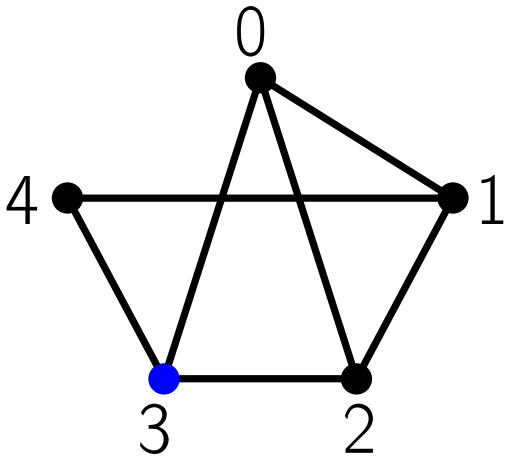

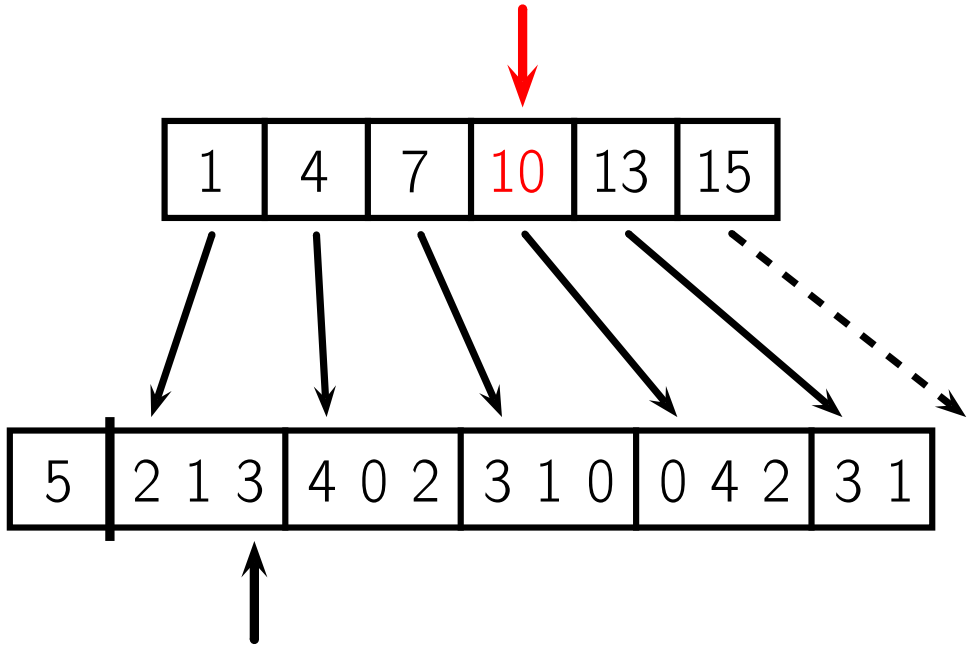

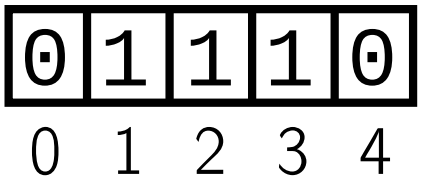

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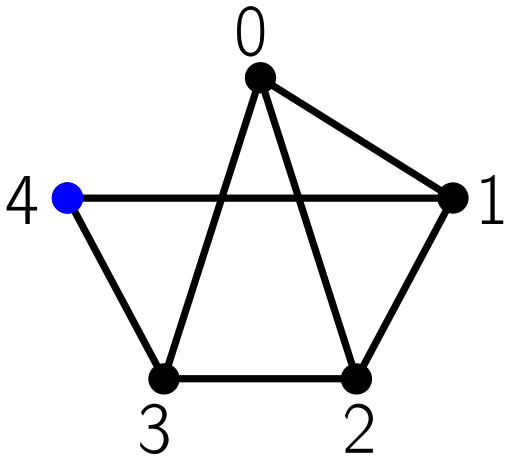

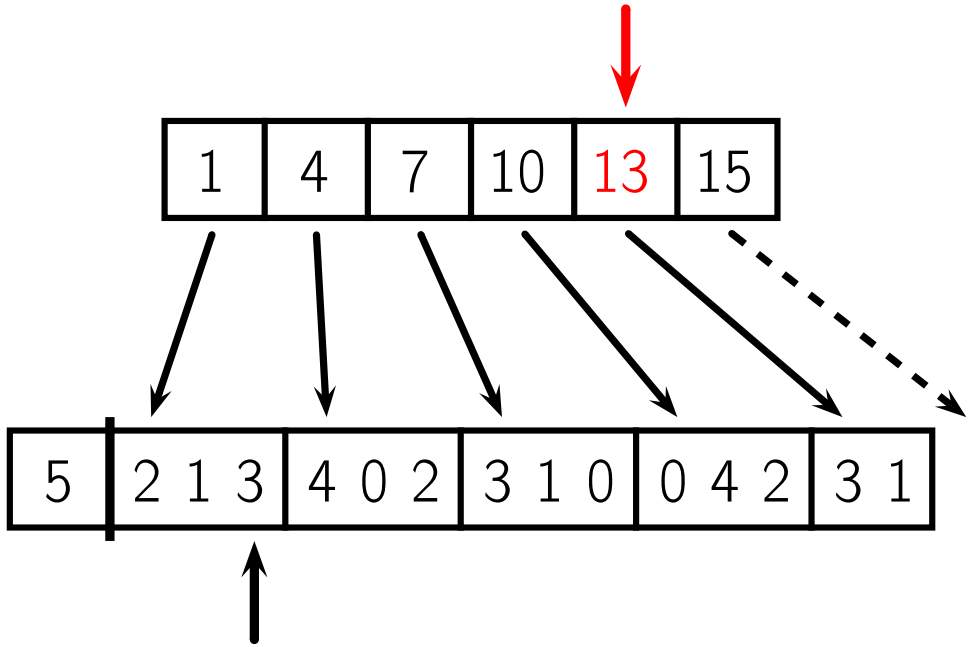

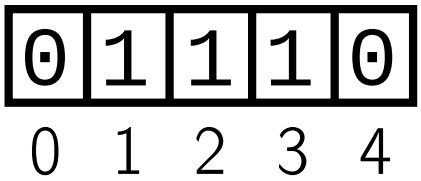

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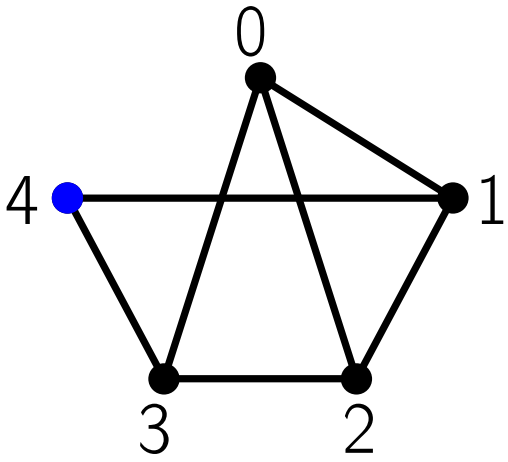

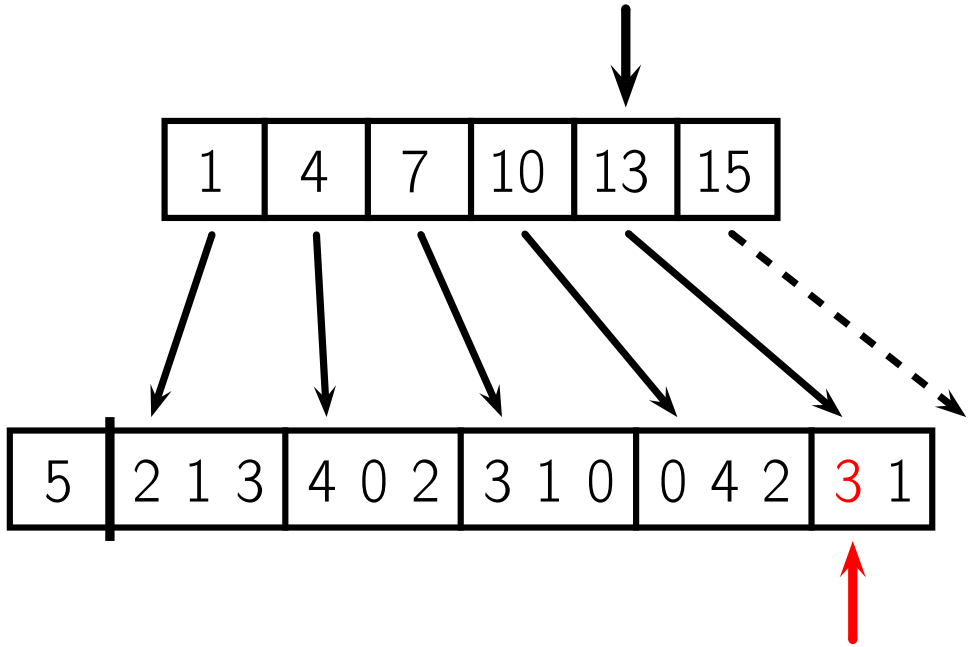

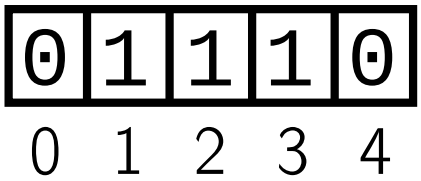

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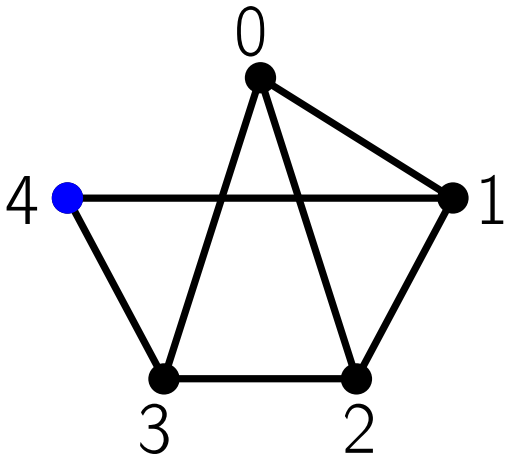

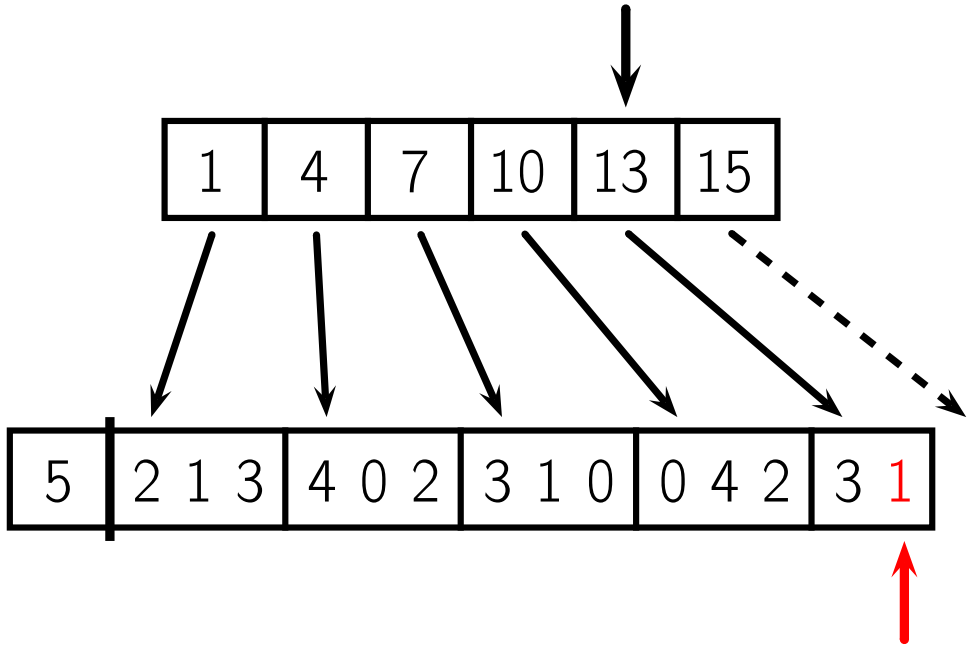

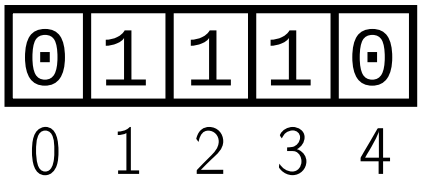

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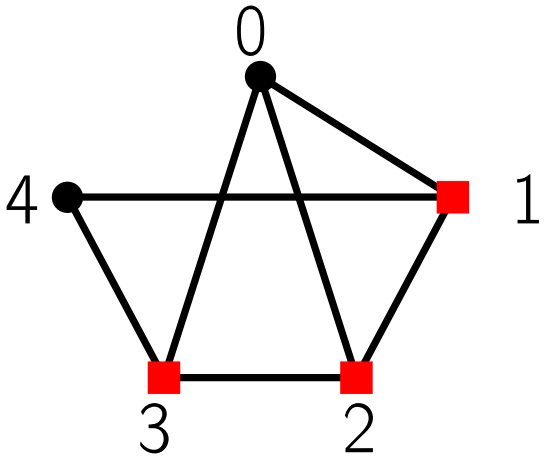

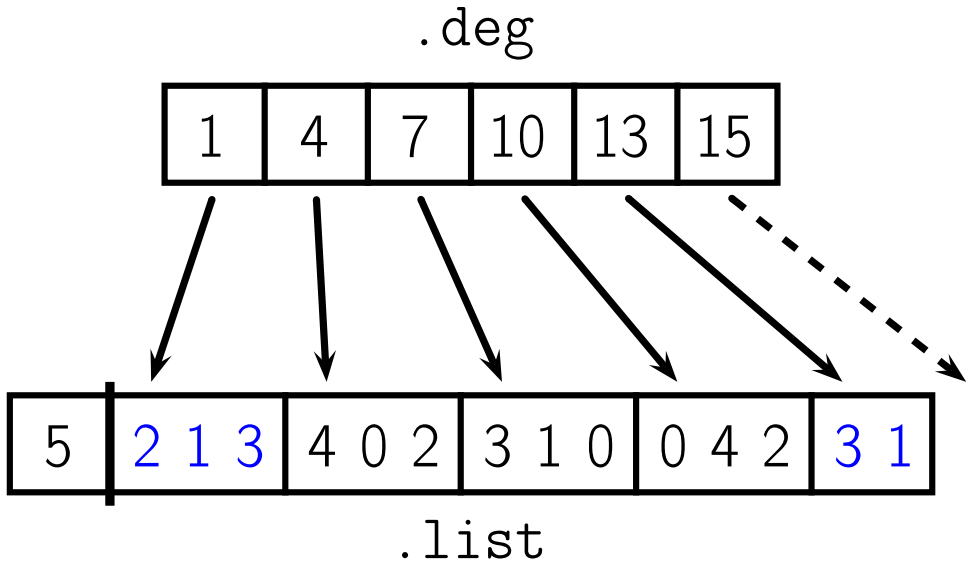

Cover produced on disk: 2, 1, 3

Number of vertices scanned in the `.list` file: five (over 14)

→ We can "step over" neighbors in the `.list` file.

Algorithms properties

Technical properties

1. `LR`, `ED` and `S-Pitt` need to allocate an n bits array.
2. `SLL` and `ASLL` need to compute degrees of neighbors.

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Approximation ratio

`LR`: Δ

`ED`: 2 (matching algorithm)

`S-Pitt`: 2 *in expectation*

L. Pitt: *A Simple Probabilistic Approximation Algorithm for Vertex Cover*, Technical Report 404, Yale, 1985

`SLL`: $\frac{\sqrt{\Delta}}{2} + \frac{3}{2}$

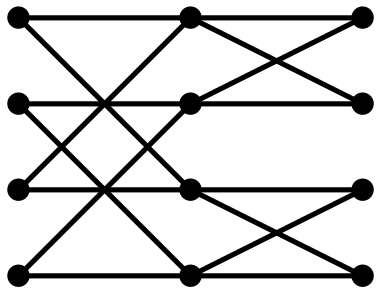
D. Avis et al.: *A List Heuristic for Vertex Cover*, ORL 2006

`LL` and `ASLL`: at least Δ

Graph families used

Sparse graphs (where $m \in \mathcal{O}(n)$):

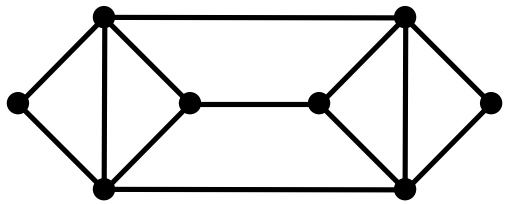
- *ButterFly*
- *de Bruijn* graphs
- grid graphs



ButterFly of dimension 2

Dense graphs (where $m \in \Theta(n^2)$):

- hypercubes
- complete bipartite graphs
- complete *split graphs*



de Bruijn of dimension 3

Why have we chosen these graphs?

1. They can be easily constructed (*with low memory capacities*).
2. Often, $OPT(G) \leq \frac{n}{2}$

Outline

Preamble

34 instances generated:

1. 13 of size $\approx 200 \cdot 10^6$ (several Gigabytes)
2. 6 of size $\approx 30 \cdot 10^9$ ($>$ 100 Gb)
3. 2 of size $\approx 100 \cdot 10^9$ (about 1.5 Tb)

Also: 13 of size $\approx$ 300,000 (several Mb on disk)

$\rightarrow$ *Random power law graphs* for low size levels

Preamble

34 instances generated:

1. 13 of size $\approx 200 \cdot 10^6$ (several Gigabytes)
2. 6 of size $\approx 30 \cdot 10^9$ ($>$ 100 Gb)
3. 2 of size $\approx 100 \cdot 10^9$ (about 1.5 Tb)

   Also: 13 of size $\approx$ 300,000 (several Mb on disk)

$\rightarrow$ *Random power law graphs* for low size levels

Evaluated criteria:

- quality of solutions
- complexity in *number of requests*

$\rightarrow$ the number of neighbors read in the `.list` file

- CPU running times

Results obtained on sparse graphs

Example with instance `butterfly-28`

n	m	OPT
7,784,628,224	15,032,385,540	3,758,096,384

Size on hard disk: 282 Gb

	Quality ($\times OPT$)	Complexity ($\times m$)	Times (CPU)
LR	**1**	0.99	$\approx$ 1h12
ED	2	**0.61**	$\approx$ 1h16
S-Pitt	1.63	0.91	$\approx$ 1h19
LL	1.93	1.03	$\approx$ 1h21
SLL	1.92	1.01	$\approx$ 5h11
ASLL	2	0.72	$\approx$ 3h39

Results obtained on dense graphs

Example with instance `compbip-35000.500000`

n	m	OPT
535,000	17,500,000,000	35,000

Size on hard disk: 261 Gb

	Quality ($\times OPT$)	Complexity ($\times m$)	Times (CPU)
LR	14.28	1	$\approx$ 24 min
ED	2	0.93	$\approx$ 23 min
S-Pitt	2.01	**0.92**	$\approx$ 22 min
LL	**1**	1.001	$\approx$ 23 min
SLL	**1**	1.001	$\approx$ 6h12
ASLL	14.28	1.002	$\approx$ 6h12

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Quality of solutions

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→ `LL` "fluctuates" more than `S-Pitt`.

`ED` and `ASLL` are overall the worst.

→ Approximation ratio of 2 often reached for `ED`

(confirmation of observations made by *F. Delbot et al.*)

- *Analytical and Experimental Comparison of Six Algorithms for the Vertex Cover*, ACM Journal of Experimental Algorithmics, 2010

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Correlation between *quality of solutions* and *number of requests*!

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Another technical aspects involved:

- hard drive access times, buffers management, etc.

$\rightarrow$ characteristics highlighted in the *I/O-efficient* model!

Limits encountered

On instance `compbip-250000.380000`, of size:

- $n = 630{,}000$ and $m = 95{,}000{,}000{,}000$ (1.41 Tb on disk),

we can run all the algorithms (except `SLL` and `ASLL`).

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But, on instance `butterfly-30`:

n	m
33,285,996,544	64,424,509,440

Size on disk: 1.17 Tb

$\rightarrow$ only `LL` can be run on our computer (4 Gb RAM)!

(CPU running times $\approx$ 5h48)

Outline

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Conclusion and perspectives

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1. Easily parallelizable $\rightarrow$ CPU running times improvement
2. $\forall$ graph, $\exists$ a vertex labeling such that LL returns OPT

We could then:

- run it several times (with different vertex labelings)

$\rightarrow$ improve quality of its results

Thanks!

Questions?